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Visibility of the lunar crescent

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SUMMARY

Prediction of the first visibility of the lunar crescent is a difficult problem involving astronomy, meteorology, and physiology. Historically, this problem has been attacked by an empirical approach where some set of observations is used to deduce a criterion for visibility. In this paper, I present a list of 201 observations and their observing circumstances for use in deriving and testing prediction algorithms. I find that criteria involving the moonset lagtime and the Moon's age are quite bad in their predictive ability. Criteria involving the relative altitude and azimuth of the Moon at sunset are better, yet still can yield incorrect predictions within a zone of uncertainty with a width of over 105 degrees in longitude. The new theoretical model of Schaefer (1988) is found to have a zone of uncertainty with an average total width of 47 degrees in longitude.

INTRODUCTION

The Islamic lunar calendar is based on the visibility of the lunar crescent, in that the first day of each month is that day after new Moon on which the crescent is first sighted at a given locality. This visibility requirement makes it difficult to predict in advance the details of the calendar. Because the ability to predict a calendar is necessary for the calendar's utility, the lunar visibility problem is a strong candidate for being the one 'non-trivial' astronomical problem that has the greatest effect on the everyday lives of the most people.

As such, the problem has long been the central focus of Islamic astronomers, especially during the classic era of Islam. The early algorithms to predict the crescent visibility were rules of thumb, usually involving the distance between the Sun and the Moon in some coordinate system. Presumably, these rules were based on some set of observations taken from one locality.

In the modern era, almost all algorithms adopt this same approach and are effectively derived from one particular set of observations. These 76 observations were originally catalogued in the *Monthly Notices of the Royal Astronomical Society* in 1910 (Fotheringham 1910) and are primarily the work of Julius Schmidt in Athens, Greece from 1859 to 1880. Fotheringham advanced a reasonable criterion for visibility based on the relative altitude and azimuth of the Moon at the time of sunset. Further refinements have been suggested by Maunder (1911), Fotheringham (1921), and Ilyas (1981, 1984).

The above approach has the severe disadvantage that the entire world is assumed to have the exact same seeing conditions as the average for the site from which the criterion was derived. This is like saying that the clarity of the air from the Amazon basin is the same as on the top of Mauna Kea. Clearly, the actual criterion for these two sites will differ greatly, yet none of the empirical algorithms allow for any variation in observing conditions. This problem is a major source of error, as I find (see below) that the difference between good conditions and fair conditions will move the location of first sighting on the Earth by over 105 degrees of longitude. This implies that over a quarter of the Earth would potentially have a date wrongly-predicted for the first day of the Islamic month. The region on Earth for which a prediction is potentially wrong is called the zone of uncertainty. A convenient way to measure the size of this zone is to quantify the longitudinal extent of the zone at, say, temperate latitudes. The ultimate goal of research on lunar visibility is to reduce the size of the zone of uncertainty as much as possible.

In an effort to improve this situation, Bruin (1977) pioneered a modern astrophysical approach. His approach was to model the reflectivity of the Moon and the actual physical processes in the human eye and the atmosphere so as to be able to calculate whether the Moon would be visible under any given conditions. Unfortunately, his work used a number of grossly incorrect assumptions, including an assumed lunar surface brightness that is many orders of magnitude in error, a twilight sky brightness that depends only on solar depression angle, physiological data that are not corrected for pupil diameter, colour, or binocular vision, and the equation of the visibility of the unevenly illuminated crescent with the visibility of a uniform circular disk 100 times smaller than the Moon. Finally, Bruin takes no account of the changing observing conditions.

I have tried to follow in Bruin's footsteps (Schaefer 1988) while using accurate physical, meteorological and physiological equations. My model predicts whether or not the Moon will be visible under any set of observing conditions. I explicitly include atmospheric clarity which is calculated from the site's altitude, latitude, temperature, relative humidity, aerosol content and the time of year. This mathematical model is combined with a lunar and solar ephemeris to yield a computer program which will predict the date of the first crescent sighting from any location.

For a test of my model and other models, I have collected 201 observations of lunar visibility from the astronomical literature. These observations and their circumstances are presented in this paper so that researchers can devise and test algorithms without having to perform exhaustive literature searches followed by extensive calculations. This listing contains 125 more observations than the last published catalogue (Fotheringham 1910) and corrects a number of calculational errors that have appeared in the literature. In addition, the calculated circumstances are presented with uniform and stated definitions.

OBSERVATIONS

The observations have been collected from the astronomical literature because this is the largest body of data available from experienced observers. Yet even this body of observations has several obviously incorrect reports. So the positive sightings of King, Willimot (Fotheringham 1921), Horner (Maunder 1911), Hoare (Ashbrook 1971), and Coleman (1932) are impossible by any reasonable criterion. In all five cases, I find that the Moon was difficult to detect on the next night, so that a simple error of date is indicated. Indeed, I personally made similar mistakes for observations 165 and 168 in placing the dates one day earlier, until careful examination of travel schedules and old personal calendars showed the earlier dates to be wrong. The negative sightings of Mommsen (numbered 73 and 74 in Fotheringham 1910) are meaningless because the attempts were made through clouds.

The specifics of all 201 observations are given in Table I. Column number 1 contains a reference number for each observation. Columns 2–4 give the year, month, and local date on which the observation was made. Column 5 indicates whether the observation was made in morning or evening twilight. Column 6 contains the Julian date of the appropriate new Moon conjunction as given by Morrison (1966). Column 7 gives the name of the observer (a † indicates that additional observers were present). Column 8 gives a number which indicates the reference for the observation, with a key being given in a footnote to the table. Column 9 gives either a 'V' if the Moon was visible or an 'I' if the Moon was not sighted. Column 10 is blank if no optical aid was used. If instead, a pair of binoculars was used to find the Moon and then the Moon was spotted with the naked eye, then an 'F' is given. If the Moon was only visible in binoculars or a telescope, then a 'B' or 'T' respectively is given. The next three columns (11–13) give the latitude (in degrees), longitude (in degrees east of Greenwich), and altitude (in feet above sea level) of the observing site. Columns 14 and 15 give the relative humidity (in per cent) and temperature (in degrees Fahrenheit) that are expected at the site at the time of observation (cf. Pearce & Smith 1984). Column 16 contains the estimated aerosol extinction coefficient (in units of hundredths of a magnitude per air mass) which includes contributions from dust, smog, and volcanic particulates. Hence observations from large cities or cities with smog problems have a coefficient which is appropriately larger (cf. Flowers, McCormick & Kurfis 1969). The volcanos considered are those of Krakatoa (1883), Mt Pelée (1902), Katmai (1912), Agung (1963), and El Chichon (1982) which are known to be the primary depositors of stratospheric dust since 1850. The volcanic component is taken to decay exponentially with a time scale of two years. The amplitude can be deduced for the various eruptions from data in Abbott *et al.* (1908–54), Angione & de Vaucouleurs (1986), Gutierrez-Moreno, Moreno & Cortes (1982), Lockwood & Thompson (1986), Mendonca, Hanson & DeLuisi (1978), Miles (1983), Przybylski (1964), and Rufener (1986). The only observations substantially affected by volcanos are observations numbers 86 and 112. The next four columns (17–20) give the arc of light, the arc of vision, the relative azimuth, and the age of the Moon for the time during twilight when the Moon is best visible on the night of observation. The time of best visibility is taken to be when the R value (see later) is at a maximum. The presentation of these results for the time of best visibility ensures that no latitude effects will be important. The arc of light is taken to be the geocentric angular distance (in degrees) between the centres of the Sun and Moon. The arc of vision is taken to be the depression angle of the Sun plus the altitude of the centre of the

TABLE I

The specifics of 201 observations of the lunar crescent (for full explanation of column headings (1)–(25), see text, pp. 513 and 519)

No. (1)	Year (2)	M (3)	D (4)	M/E (5)	JD (conj.) (6)	Observer (7)	RF* (8)	V/I (9)	B/F (10)	Lat. (11)	Long. (12)	Alt. (13)	RH (14)	*T* (15)	k_D (16)	ARCL (17)	ARCV (18)	DAZ (19)	Age (20)	Lag (21)	*R DR* (22) (23)	Y/N (24)	Sig. (25)
1	1859	7	1	E	2 400 226·112	Schmidt	1	V		38·0	23·7	400	35	85	5	16·3	12·1	11·3	27·9	71	0·8±0·3	Y	(3)
2	1859	10	27	E	2 400 343·523	Schmidt	1	V		38·0	23·7	400	55	70	5	21·5	6·8	20·5	39·3	78	−0·3±0·4	N	(−1)
3	1860	1	23	E	2 400 432·511	Schmidt	1	I		38·0	23·7	400	65	50	5	7·1	6·4	4·1	15·6	29	−1·9±0·2	Y	(8)
4	1860	2	23	E	2 400 462·318	Schmidt	1	V		38·0	23·7	400	60	55	5	20·6	20·5	3·3	45·4	69	2·6±0·1	Y	( )
5	1860	6	20	E	2 400 580·725	Schmidt	1	V		38·0	23·7	400	40	80	5	20·1	14·6	14·1	37·3	87	1·8±0·2	Y	(8)
6	1861	3	12	E	2 400 846·067	Schmidt	1	V		38·0	23·7	400	55	55	5	13·3	13·2	−3·0	27·6	37	1·1±0·2	Y	(7)
7	1861	8	7	E	2 400 994·038	Schmidt	1	I		38·0	23·7	400	35	85	5	16·2	5·3	15·6	28·6	56	−1·6±0·4	Y	(4)
8	1861	8	8	E	2 400 994·038	Schmidt	1	V		38·0	23·7	400	35	85	5	29·4	10·8	27·5	53·1	104	1·6±0·3	Y	(5)
9	1861	9	7	E	2 401 023·426	Schmidt	1	V		38·0	23·7	400	45	80	5	38·7	13·7	36·4	67·2	135	2·5±0·3	Y	(9)
10	1861	10	5	E	2 401 052·790	Schmidt	1	I		38·0	23·7	400	55	70	5	20·1	5·4	19·5	33·2	67	−1·0±0·4	Y	(3)
11	1861	11	4	E	2 401 081·171	Schmidt	1	V		38·0	23·7	400	65	60	5	28·0	14·4	24·3	48·0	113	2·5±0·2	Y	( )
12	1861	12	3	E	2 401 111·596	Schmidt	1	V		38·0	23·7	400	65	55	5	21·4	15·0	15·5	37·6	92	2·2±0·1	Y	( )
13	1862	1	1	E	2 401 141·079	Schmidt	1	V		37·9	22·9	400	65	50	5	14·6	13·2	6·7	26·2	60	1·2±0·2	Y	(8)
14	1862	3	31	E	2 401 229·823	Schmidt	1	V		38·0	23·7	400	55	55	5	16·2	16·2	2·6	33·8	51	1·9±0·1	Y	( )
15	1862	4	29	E	2 401 259·475	Schmidt	1	I		38·0	23·7	400	50	65	5	8·8	8·8	2·6	18·3	28	−0·9±0·3	Y	(3)
16	1862	7	28	E	2 401 348·379	Schmidt	1	I		38·0	23·7	400	35	85	5	22·5	8·3	21·1	44·9	81	0·3±0·3	N	(−1)
17	1864	1	10	E	2 401 879·825	Schmidt	1	V		38·0	23·7	400	65	50	5	19·5	18·9	5·5	32·5	73	2·4±0·1	Y	( )
18	1864	2	8	E	2 401 909·257	Johnson	2	V		53·5	−2·3	600	80	40	10	14·8	14·2	4·7	23·8	46	1·2±0·2	Y	(6)
19	1864	3	9	E	2 401 938·665	Schmidt	1	V		38·0	23·7	400	55	55	5	21·7	21·5	3·4	37·3	73	2·8±0·1	Y	( )
20	1864	5	6	E	2 401 997·508	Schmidt	1	I		39·6	26·2	400	50	75	5	9·2	7·8	5·4	17·3	37	−1·5±0·4	Y	(4)
21	1864	6	6	E	2 402 026·986	Schmidt	1	V		38·0	23·7	400	40	80	5	26·9	17·0	21·0	54·9	113	2·7±0·2	Y	( )
22	1864	8	4	E	2 402 086·108	Schmidt	1	I		38·0	23·7	400	35	85	5	24·0	8·4	22·6	51·3	84	0·5±0·3	N	(−1)
23	1864	9	3	E	2 402 115·755	Schmidt	1	V		38·0	23·7	400	45	80	5	27·2	10·5	25·3	59·3	96	1·4±0·3	Y	(4)
24	1864	11	1	E	2 402 175·145	Schmidt	1	V		38·0	23·7	400	65	60	5	23·7	15·3	18·4	48·7	97	2·4±0·2	Y	( )
25	1865	1	28	E	2 402 263·896	Schmidt	1	V		38·0	23·7	400	65	50	5	18·2	18·1	34	31·1	63	2·2±0·1	Y	( )
26	1865	3	28	E	2 402 322·728	Schmidt	1	V		38·0	23·7	400	55	55	5	21·1	20·4	6·3	36·1	78	2·7±0·1	Y	( )
27	1865	4	26	E	2 402 352·092	Schmidt	1	V		38·0	23·7	400	50	65	5	16·1	13·9	8·5	27·7	65	1·4±0·2	Y	(8)
28	1865	6	24	E	2 402 410·831	Schmidt	1	I		38·0	23·7	400	40	80	5	18·6	8·8	16·6	34·3	74	0·1±0·3	N	(−0)
29	1865	7	24	E	2 402 440·270	Schmidt	1	V		38·0	23·7	400	35	85	5	23·7	9·4	22·0	47·7	87	0·8±0·3	Y	(2)
30	1865	10	21	E	2 402 529·186	Schmidt	1	V		38·0	23·7	400	55	70	5	21·8	13·8	17·2	47·9	87	1·9±0·2	Y	(8)
31	1866	1	17	E	2 402 618·359	Schmidt	1	I		38·0	23·7	400	65	50	5	11·0	11·0	2·6	19·5	36	0·2±0·2	N	(−1)
32	1866	4	16	E	2 402 706·794	Schmidt	1	V		38·0	23·7	400	50	65	5	20·6	18·2	10·2	34·8	83	2·4±0·1	Y	( )
33	1867	2	5	E	2 403 002·261	Schmidt	1	I		38·0	23·7	400	60	55	5	11·0	10·9	2·9	22·2	38	0·2±0·2	N	(−1)
34	1867	11	27	E	2 403 296·716	Schmidt	1	V		38·0	23·7	400	65	60	5	16·7	14·4	8·9	34·7	70	1·6±0·2	Y	(9)
35	1868	6	22	E	2 403 504·115	Schmidt	1	V		38·0	23·7	400	40	80	5	30·0	18·1	24·2	52·0	125	2·9±0·2	Y	( )
36	1869	5	12	E	2 403 829·172	Schmidt	1	I		38·0	23·7	400	50	75	5	13·6	9·2	10·4	25·7	56	−0·4±0·3	Y	(1)
37	1870	7	25	M	2 404 271·971	Schmidt	1	V		38·0	23·7	400	50	75	5	40·0	31·6	25·1	−80·8	170	3·7±0·1	Y	( )
38	1871	2	20	E	2 404 478·075	Schmidt	1	V		38·0	23·7	400	60	55	5	14·5	11·9	8·8	27·0	59	0·9±0·2	Y	(5)
39	1871	4	20	E	2 404 537·295	Schmidt	1	I		38·0	23·7	400	50	65	5	11·0	8·4	7·6	22·4	44	−0·7±0·3	Y	(3)
40	1871	5	20	E	2 404 566·948	Schmidt	1	I		38·0	23·7	400	50	75	5	14·1	11·1	9·1	31·4	60	0·4±0·3	N	(−1)

41	1871	6	18	E	2 404 596·604	Schmidt	1	I	38·0	23·7	400	40	80	5	7·2	5·6	5·3	15·4	31	−2·6±0·3	Y	(7)
42	1871	6	19	E	2 404 596·604	Schmidt	1	V	38·0	23·7	400	40	80	5	18·2	13·6	12·4	40·1	79	1·5±0·2	Y	(6)
43	1871	8	17	E	2 404 655·793	Schmidt	1	V	38·0	23·7	400	35	85	5	18·1	12·7	13·1	34·9	73	1·2±0·3	Y	(4)
44	1871	9	14	M	2 404 685·300	Schmidt	1	V	38·0	23·7	400	60	70	5	9·3	9·0	−3·6	−15·4	22	−1·0±0·4	N	(−3)
45	1872	6	7	E	2 404 950·642	Schmidt	1	V	38·0	23·7	400	40	80	5	17·9	14·7	10·5	39·2	77	1·7±0·2	Y	(8)
46	1872	7	6	E	2 404 980·267	Schmidt	1	I	38·0	23·7	400	35	85	5	11·4	9·5	6·7	23·9	48	−0·5±0·4	Y	(1)
47	1872	9	4	E	2 405 039·537	Schmidt	1	V	38·0	23·7	400	45	80	5	19·8	13·2	15·1	40·6	79	1·5±0·3	Y	(5)
48	1872	9	30	M	2 405 069·146	Schmidt	1	V	38·0	23·7	400	60	70	5	28·9	28·9	3·5	−59·9	98	3·4±0·1	Y	( )
49	1872	10	3	E	2 405 069·146	Schmidt	1	I	38·0	23·7	400	55	70	5	12·9	9·1	9·5	25·0	51	−0·4±0·3	Y	(1)
50	1872	10	4	E	2 405 069·146	Schmidt	1	V	38·0	23·7	400	55	70	5	24·6	14·4	20·2	49·2	95	2·2±0·2	Y	( )
51	1872	12	31	E	2 405 157·775	Schmidt	1	V	38·0	23·7	400	65	55	5	19·6	13·7	14·3	33·5	85	1·8±0·2	Y	( )
52	1873	4	27	E	2 405 275·446	Schmidt	1	I	38·0	23·7	400	50	65	5	10·3	9·3	5·1	19·0	41	−0·6±0·3	Y	(2)
53	1873	5	27	E	2 405 304·889	Schmidt	1	V	38·0	23·7	400	50	75	5	16·8	14·9	8·1	33·1	70	1·5±0·2	Y	(7)
54	1873	12	20	E	2 405 512·284	Schmidt	1	I	38·0	23·7	400	65	55	5	11·7	5·4	10·7	20·5	49	−1·7±0·3	Y	(6)
55	1874	4	17	E	2 405 630·078	Schmidt	1	V	38·0	23·7	400	50	65	5	16·3	15·5	5·6	27·9	62	1·7±0·2	Y	( )
56	1875	6	4	E	2 406 043·431	Denning	2	V	51·5	−2·6	0	75	65	5	14·1	11·3	8·8	22·9	57	0·2±0·3	Y	(1)
57	1875	7	4	E	2 406 072·726	Schmdit	1	V	38·0	23·7	400	35	85	5	21·4	16·5	14·0	37·3	93	2·2±0·2	Y	( )
58	1876	2	26	E	2 406 309·765	Schmidt	1	V	38·0	23·7	400	60	55	5	17·3	16·5	5·7	34·7	64	2·0±0·1	Y	( )
59	1876	6	22	E	2 406 427·429	Schmidt	1	I	38·0	23·7	400	40	80	5	12·8	11·6	6·1	20·2	54	0·3±0·3	N	(−1)
60	1876	7	22	E	2 406 456·704	Schmidt	1	V	38·0	23·7	400	35	85	5	21·9	13·9	17·1	37·6	91	1·8±0·2	Y	(7)
61	1877	3	16	E	2 406 693·621	Schmidt	1	V	38·0	23·7	400	55	55	5	18·5	18·4	3·2	38·5	62	2·3±0·1	Y	( )
62	1877	6	12	E	2 406 782·105	Schmidt	1	V	38·0	23·7	400	40	80	5	16·3	14·3	8·2	28·1	69	1·4±0·2	Y	(6)
63	1877	11	7	E	2 406 928·867	Schmidt	1	V	38·0	23·7	400	65	60	5	29·5	13·3	26·5	55·2	118	2·4±0·2	Y	( )
64	1877	12	6	E	2 406 958·421	Schmidt	1	V	38·0	23·7	400	65	55	5	21·1	12·2	17·4	41·7	90	1·7±0·2	Y	(8)
65	1878	1	5	E	2 406 988·085	Schmidt	1	V	38·0	23·7	400	65	50	5	23·4	19·5	13·2	50·2	99	2·8±0·1	Y	( )
66	1878	6	2	E	2 407 136·575	Schmidt	1	V	38·0	23·7	400	40	80	5	21·0	18·2	10·8	40·8	89	2·4±0·2	Y	( )
67	1878	7	1	E	2 407 166·021	Schmidt	1	V	38·0	23·7	400	35	85	5	16·1	11·6	11·5	30·0	70	0·7±0·3	Y	(2)
68	1878	7	31	E	2 407 195·402	Schmidt	1	V	38·0	23·7	400	35	85	5	25·2	11·7	22·5	44·5	95	1·6±0·3	Y	(5)
69	1878	10	27	E	2 407 283·458	Schmidt	1	V	38·0	23·7	400	55	70	5	24·3	8·8	22·8	41·0	90	0·6±0·4	Y	(2)
70	1878	11	26	E	2 407 312·882	Schmidt	1	V	38·0	23·7	400	65	60	5	30·3	18·5	24·3	54·8	130	3·1±0·1	Y	( )
71	1879	5	23	E	2 407 490·744	Schmidt	1	V	38·0	23·7	400	50	75	5	28·1	24·1	14·6	60·7	120	3·2±0·1	Y	( )
72	1879	6	22	E	2 407 520·347	Schmidt	1	V	38·0	23·7	400	40	80	5	34·1	21·9	26·4	70·4	144	3·4±0·1	Y	( )
73	1879	7	22	E	2 407 549·879	Schmidt	1	V	38·0	23·7	400	35	85	5	42·1	19·1	37·8	81·5	159	3·3±0·1	Y	( )
74	1879	12	11	M	2 407 696·962	Mommsen	1	V	38·0	23·7	400	80	45	5	32·0	24·4	21·0	−54·4	138	3·5±0·1	Y	( )
75	1879	12	15	E	2 407 696·962	Mommsen	1	V	38·0	23·7	400	65	55	5	30·1	23·6	19·1	53·0	129	3·4±0·1	Y	( )
76	1880	1	10	M	2 407 726·444	Mommsen	1	V	38·0	23·7	400	80	45	5	23·5	15·3	18·1	−41·6	102	2·4±0·1	Y	( )
77	1881	3	30	E	2 408 169·439	Denning	2	V	51·5	−2·6	0	80	45	6	11·8	11·6	3·1	20·9	34	0·4±0·2	Y	(2)
78	1889	11	22	M	2 411 329·571	Degroupet	3	V	50·9	4·2	100	85	45	6	11·6	10·8	5·0	−19·1	40	0·1±0·2	Y	(0)
79	1899	3	12	E	2 414 725·328	Schoch	2	V	52·5	13·3	200	70	45	6	13·3	13·2	3·1	22·0	37	1·0±0·2	Y	(6)
80	1900	5	29	E	2 415 168·118	Johnson	3	V	38·7	−0·7	1000	60	70	6	15·3	11·5	10·5	29·1	66	0·6±0·3	Y	(2)
81	1901	4	19	E	2 415 493·401	Johnson	3	V	50·7	−2·8	100	70	65	6	13·2	11·5	7·0	22·3	49	0·2±0·3	Y	(1)
82	1908	2	3	E	2 417 973·858	Henderson	2	V	56·0	−3·2	0	80	40	9	19·6	12·3	15·5	33·2	80	1·4±0·2	Y	(6)
83	1909	2	21	E	2 418 357·953	Gheury	3	V	51·1	0·0	100	75	40	11	19·1	13·5	13·8	31·4	76	1·5±0·2	Y	(6)

TABLE I. (*continued*)

No. (1)	Year (2)	M (3)	D (4)	M/E (5)	JD (conj.) (6)	Observer (7)	RF* (8)	V/I (9)	B/F (10)	Lat. (11)	Long. (12)	Alt. (13)	RH (14)	*T* (15)	k_D (16)	ARCL (17)	ARCV (18)	DAZ (19)	Age (20)	Lag (21)	*R* *DR* (22) (23)	Y/N (24)	Sig. (25)
84	1911	6	27	E	2419214·055	Bougon	4	V		49·9	2·3	50	60	70	6	19·1	12·4	14·8	31·7	83	1·1 ± 0·3	Y	(4)
85	1911	8	25	E	2419272·672	Bougon	4	V		49·9	2·3	50	65	70	6	21·4	9·2	19·6	39·2	84	0·3 ± 0·4	Y	(1)
86	1913	11	28	E	2420099·570	Long†	5	V		−33·9	18·5	300	60	70	10	10·2	10·2	−2·6	16·5	35	−0·8 ± 0·5	N	(−2)
87	1915	3	16	E	2420572·321	Schoch	2	V		49·4	8·7	700	65	45	6	11·1	11·0	3·0	22·5	33	0·2 ± 0·2	Y	(1)
88	1916	4	3	E	2420956·181	Schoch	2	V		49·4	8·7	700	60	50	6	14·1	14·0	3·2	26·5	41	1·3 ± 0·2	Y	(8)
89	1918	3	13	E	2421665·328	Schoch	2	V		50·2	5·1	300	80	50	6	14·2	14·1	3·1	22·6	41	1·2 ± 0·2	Y	(6)
90	1919	4	1	E	2422049·378	Whitmell	2	V		53·9	−1·6	600	60	50	9	13·4	12·8	4·9	22·4	43	0·7 ± 0·2	Y	(3)
91	1921	2	8	E	2422728·525	Campbell†	2	V		42·3	−71·1	100	60	35	6	11·0	11·0	2·6	22·2	33	0·4 ± 0·2	Y	(2)
92	1921	2	8	E	2422728·525	Lampland†	2	V		35·2	−111·7	8100	50	40	5	12·3	12·2	−2·8	25·1	38	1·1 ± 0·1	Y	(9)
93	1921	2	8	E	2422728·525	Sykes	2	V		32·2	−110·0	8010	55	40	5	12·2	12·1	−3·1	25·0	38	1·1 ± 0·1	Y	(9)
94	1921	2	8	E	2422728·525		2	V	F	34·2	−118·0	6600	55	65	6	12·4	12·4	−2·8	25·5	39	0·8 ± 0·2	Y	(4)
95	1921	2	8	E	2422728·525		2	I		36·5	−6·2	0	70	60	6	9·3	9·2	−3·1	17·8	25	−0·9 ± 0·3	Y	(3)
96	1921	2	8	E	2422728·525		2	I		38·8	−9·1	0	65	55	6	9·4	9·3	−2·9	18·0	25	−0·7 ± 0·3	Y	(3)
97	1921	8	4	E	2422905·345	MacKenzie	5	V		−33·9	18·5	100	70	60	7	12·7	12·7	2·6	20·5	41	0·6 ± 0·3	Y	(2)
98	1921	10	31	E	2422933·485	MacKenzie	6	I		−33·9	18·5	100	60	65	7	9·9	8·3	−6·0	17·9	40	−1·2 ± 0·4	Y	(3)
99	1921	12	30	E	2423052·735	MacKenzie	6	I		−33·9	18·5	100	60	75	7	18·0	9·2	−15·7	36·8	70	−0·1 ± 0·5	Y	(0)
100	1922	1	29	E	2423082·492	MacKenzie	6	I		−33·9	18·5	100	60	75	9	19·6	8·9	−17·6	42·4	71	−0·1 ± 0·6	Y	(0)
101	1922	2	28	E	2423112·283	MacKenzie	6	V		−33·9	18·5	100	60	75	6	21·2	10·8	−18·5	47·1	77	0·9 ± 0·4	Y	(2)
102	1922	3	29	E	2423142·044	MacKenzie	6	I		−33·9	18·5	100	60	75	6	13·0	8·0	−10·6	28·0	49	−1·0 ± 0·5	Y	(2)
103	1922	4	27	E	2423171·710	MacKenzie	6	I		−33·9	18·5	100	65	70	9	6·3	5·6	−3·9	11·3	25	−3·0 ± 0·5	Y	(6)
104	1922	4	28	E	2432171·710	MacKenzie	6	V		−33·9	18·5	100	65	70	6	17·7	13·7	−11·5	35·8	73	1·4 ± 0·3	Y	(4)
105	1922	5	27	E	2423201·253	MacKenzie	6	V		−33·9	18·5	100	70	65	6	12·4	11·6	−5·2	22·3	51	0·2 ± 0·3	Y	(1)
106	1931	8	13	M	2426567·352	Danjon	7	V	T	52·6	13·9	150	60	70	6	10·9	10·4	4·1	−17·3	35	0·3 ± 0·4	Y	(1)
107	1942	12	8	E	2430701·582	Oravec	8	V	F	40·7	−74·0	0	65	35	11	12·3	11·0	6·1	20·1	50	0·1 ± 0·3	Y	(1)
108	1953	4	14	E	2434481·340	Meuss	8	V	F	51·1	5·3	100	75	55	6	14·1	13·8	4·0	23·3	43	1·1 ± 0·2	Y	(5)
109	1954	3	5	E	2434806·633	Quigley	7	V		44·5	−88·0	800	65	40	6	13·1	13·1	2·6	21·3	38	1·0 ± 0·1	Y	(7)
110	1961	1	17	E	2437316·396	McClure	8	V		34·0	−118·3	1000	50	60	6	16·7	16·1	4·9	28·4	65	1·8 ± 0·1	Y	( )
111	1962	4	5	E	2437759·323	Thackeray	7	V	F	−25·8	−28·2	4500	50	70	8	15·3	13·4	−7·9	24·6	61	1·2 ± 0·3	Y	(4)
112	1965	9	24	M	2439028·637	Meuss	8	V		51·1	5·3	100	75	65	11	13·7	13·6	3·1	−22·5	40	0·6 ± 0·4	Y	(1)
113	1970	4	6	E	2440682·674	Quigley	7	V	F	44·5	−88·0	800	65	50	6	12·1	12·0	2·8	20·9	37	0·6 ± 0·2	Y	(3)
114	1970	4	6	E	2440682·674	Penegor	7	V		48·0	−122·0	500	60	55	6	13·3	13·1	3·6	23·4	42	0·9 ± 0·2	Y	(5)
115	1970	6	4	E	2440741·239	Fleming	8	V		26·3	−98·2	0	60	85	6	12·3	12·3	2·6	32·2	48	−0·1 ± 0·6	N	(−0)
116	1970	6	4	E	2440741·239	Hoffler	8	V		28·0	−97·0	100	60	85	6	12·3	12·3	2·8	32·2	48	−0·1 ± 0·6	N	(−0)
117	1971	3	27	E	2441038·308		8	V		51·0	0·0	0	70	45	6	14·7	14·4	4·0	−0·1	45	1·4 ± 0·1	Y	(9)
118	1971	4	25	E	2441066·668	Pence	7	V	F	39·5	−88·2	600	60	60	6	13·3	13·3	2·6	21·3	42	0·9 ± 0·2	Y	(4)
119	1972	3	15	E	2441391·983	Moran	9	V	B	35·5	−117·6	3700	20	70	6	9·8	9·4	−3·6	14·9	25	0·0 ± 0·2	Y	(0)
120	1972	3	15	E	2441391·983	McMahon†	9	I	B	35·5	−117·6	3000	20	70	6	9·8	9·4	−3·6	14·9	25	0·0 ± 0·2	Y	(0)
121	1973	3	5	E	2441746·501	Hanford†	10	V	F	40·0	−85·0	1000	70	45	6	13·6	13·5	−2·8	24·3	39	1·1 ± 0·2	Y	(7)
122	1973	7	1	E	2441863·986	Austin	11	V	B	−44·0	170·5	3900	80	50	6	10·6	8·6	−6·8	18·0	44	−0·3 ± 0·3	N	(−1)
123	1976	12	21	E	2443133·590	Olsen	12	V	F	42·0	−91·6	900	70	30	6	12·9	12·0	5·2	21·2	50	0·9 ± 0·1	Y	(6)

124	1976	12	21	E	2 443 133·590	Jones†	12	V		29·9	−81·3	0	70	75	5	12·8	12·7	3·1	20·9	49	0·3±0·4	Y	(1)
125	1976	12	21	E	2 443 133·590	Patterson	13	V		37·6	−122·5	0	50	65	6	14·1	13·4	4·9	23·5	55	1·0±0·2	Y	(5)
126	1976	12	21	E	2 443 133·590	Victor†	14	V		42·7	−83·6	700	80	30	7	12·5	11·7	5·2	20·6	48	0·7±0·2	Y	(4)
127	1977	2	18	E	2 443 192·652	Schenk	12	V	F	43·8	−87·7	100	70	30	6	10·8	10·8	2·7	20·4	33	0·3±0·2	Y	(2)
128	1977	11	10	M	2 443 458·799	Patterson†	15	V	F	30·7	−104·0	7900	40	60	5	10·9	10·9	2·7	−18·3	39	0·5±0·2	Y	(3)
129	1977	12	11	E	2 443 488·232	Keszthelyi	15	V		47·8	20·0	600	85	35	6	13·7	11·6	7·8	22·0	55	0·7±0·2	Y	(4)
130	1978	1	9	E	2 443 517·667	McGraw	16	V		41·6	−93·6	800	70	25	6	12·4	12·1	3·6	19·8	44	0·8±0·1	Y	(6)
131	1978	1	9	E	2 443 517·667	Korycansky	15	V	B	38·9	−76·9	100	60	40	6	11·8	11·7	3·0	18·7	41	0·9±0·2	Y	(5)
132	1978	1	9	E	2 443 517·667	Hill	15	V	F	36·0	−79·8	1000	70	45	6	12·0	12·0	2·8	19·0	42	0·6±0·2	Y	(3)
133	1978	1	9	E	2 443 517·667	Talbert	15	V		36·0	−79·8	1000	70	45	6	12·0	12·0	2·8	19·0	42	0·6±0·2	Y	(3)
134	1978	1	9	E	2 443 517·667	Connor	15	V	F	36·0	−79·8	1000	70	45	6	12·0	12·0	2·8	19·0	42	0·6±0·2	Y	(2)
135	1978	1	9	E	2 443 517·667	Watts	15	V	F	34·0	−81·1	500	70	55	5	12·1	12·1	2·6	19·2	43	0·5±0·2	Y	(2)
136	1978	1	9	E	2 443 517·667	Fleming	15	V	F	34·0	−81·1	500	70	55	5	12·1	12·1	2·6	19·2	43	0·5±0·2	Y	(2)
137	1978	1	9	E	2 443 517·667	Faber	15	V	F	33·9	−84·3	1100	70	45	6	12·2	12·2	2·7	19·4	43	0·7±0·2	Y	(4)
138	1978	1	9	E	2 443 517·667	Barrett	15	V		41·9	−87·6	800	70	30	6	12·1	11·9	3·5	19·3	43	0·7±0·1	Y	(5)
139	1978	1	9	E	2 443 517·667	Bricker†	15	V	F	27·7	−82·7	0	70	70	6	12·3	12·3	−2·7	19·4	43	0·2±0·4	Y	(0)
140	1978	1	9	E	2 443 517·667	Guzman†	15	V		29·9	−81·3	0	70	70	6	12·2	12·2	−2·6	19·3	43	0·1±0·4	Y	(0)
141	1978	1	9	E	2 443 517·667	Morris	15	V		29·9	−81·3	0	70	70	6	12·2	12·2	−2·6	19·3	43	0·1±0·4	Y	(0)
142	1978	1	9	E	2 443 517·667	Bohn	15	V	F	43·0	−89·8	900	70	30	6	12·2	11·9	3·7	19·4	43	0·7±0·1	Y	(5)
143	1978	1	9	E	2 443 517·667	Rosamond†	15	V	F	30·0	−90·2	0	70	60	6	12·5	12·5	−2·6	19·9	44	0·5±0·3	Y	(2)
144	1978	1	9	E	2 443 517·667	Schiffer	15	V	F	30·0	−90·2	0	70	60	6	12·5	12·5	−2·6	19·9	44	0·5±0·3	Y	(2)
145	1978	1	9	E	2 443 517·667	Meibaum†	15	V		30·0	−90·2	0	70	60	6	12·5	12·5	−2·6	19·9	44	0·5±0·3	Y	(2)
146	1978	1	9	E	2 443 517·667	Ikels†	15	V		29·7	−98·1	500	70	60	6	12·8	12·8	−2·6	20·5	46	0·6±0·3	Y	(3)
147	1978	3	9	E	2 443 576·609	Bishop	16	V		45·1	−64·2	30	60	35	4	10·8	10·2	4·4	20·2	38	0·2±0·1	Y	(2)
148	1978	3	9	E	2 443 576·609	Bortle	16	V	F	42·7	−73·8	600	65	40	6	11·1	10·7	4·2	20·9	40	0·2±0·2	Y	(1)
149	1978	3	9	E	2 443 576·609	Piaruli	16	V		41·3	−72·9	200	55	45	6	11·1	10·7	4·0	20·8	40	0·2±0·2	Y	(1)
150	1978	3	9	E	2 443 576·609	Dessert	16	V		42·7	−73·8	600	65	40	6	11·1	10·7	4·2	20·9	40	0·2±0·2	Y	(1)
151	1978	3	9	E	2 443 576·609	Cebula	16	V		40·5	−89·0	800	65	40	6	11·7	11·3	4·0	21·9	42	0·5±0·2	Y	(3)
152	1978	3	9	E	2 443 576·609	McGraw	16	V		41·6	−93·6	800	60	45	6	11·8	11·4	4·2	22·2	42	0·5±0·2	Y	(3)
153	1978	3	9	E	2 443 576·609	Newman	16	V		41·6	−93·6	800	60	45	6	11·8	11·4	4·2	22·2	42	0·5±0·2	Y	(3)
154	1978	3	9	E	2 443 576·609	Phelps	16	V		40·5	−89·0	800	65	40	6	11·7	11·3	4·0	21·9	42	0·5±0·2	Y	(3)
155	1978	3	9	E	2 443 576·609	Haasdyk	16	V		50·3	−119·3	2000	65	35	6	12·7	11·4	6·1	24·0	46	0·7±0·2	Y	(5)
156	1978	3	9	E	2 443 576·609	Dombrowski	17	V	F	41·3	−72·9	100	60	40	6	11·1	10·7	4·0	20·8	40	0·2±0·2	Y	(1)
157	1979	1	27	M	2 443 901·778	Skiff†	18	V	B	35·2	−111·7	8100	50	40	5	10·4	9·2	5·4	−16·5	42	0·3±0·2	Y	(2)
158	1979	1	28	E	2 443 901·778	ACAC	18	V		29·9	−81·3	0	70	70	6	10·4	10·4	2·6	16·7	37	−0·7±0·4	N	(−2)
159	1979	1	28	E	2 443 901·778	Sherman†	18	V	B	29·7	−82·4	0	70	70	6	10·5	10·5	2·6	16·8	37	−0·3±0·4	N	(−1)
160	1979	1	28	E	2 443 901·778	Olsen	18	V	B	42·0	−91·7	800	70	25	6	10·6	10·3	3·5	17·2	37	0·4±0·2	Y	(2)
161	1979	1	28	E	2 443 901·778	Fefferman	18	V	B	38·7	−90·3	600	65	35	6	10·6	10·5	3·1	17·2	37	0·4±0·2	Y	(2)
162	1979	1	28	E	2 443 901·778	Verne†	18	V		42·0	−93·6	900	70	25	6	10·6	10·4	3·5	17·3	38	0·1±0·2	Y	(1)
163	1979	1	28	E	2 443 901·778	Peterson	18	V	F	47·6	−122·3	100	80	40	6	11·6	10·9	4·8	19·1	42	0·2±0·2	Y	(1)
164	1979	1	28	E	2 443 901·778	Westfall	18	V	F	37·8	−122·4	200	70	50	6	11·8	11·6	3·3	19·4	43	0·4±0·2	Y	(2)
165	1980	7	13	E	2 444 432·782	Schaefer†	17	V		41·4	−70·7	0	70	75	6	20·8	10·6	18·1	42·0	85	0·6±0·5	Y	(1)
166	1981	7	30	M	2 444 816·661	Schaefer	17	I		42·3	−71·3	100	70	65	7	10·3	8·4	6·4	−18·5	42	−1·3±0·4	Y	(3)

TABLE I. (*continued*)

No. (1)	Year (2)	M (3)	D (4)	M/E (5)	JD (conj.) (6)	Observer (7)	RF* (8)	V/I (9)	B/F (10)	Lat. (11)	Long. (12)	Alt. (13)	RH (14)	*T* (15)	k_{D} (16)	ARCL (17)	ARCV (18)	DAZ (19)	Age (20)	Lag (21)	*R* *DR* (22) (23)	Y/N (24)	Sig. (25)
167	1985	1	21	E	2 446 086·604	O'Meara	17	V		19·0	−155·0	13 600	30	40	4	13·8	12·5	6·5	26·2	58	1·6±0·1	Y	( )
168	1986	10	5	E	2 446 707·288	Schaefer†	17	V		40·8	−73·2	100	60	65	9	28·5	11·4	26·3	52·2	105	1·6±0·4	Y	(4)
169	1986	12	31	E	2 446 795·632	Schaefer†	17	I	B	39·0	−77·0	100	70	45	11	12·4	5·9	11·2	18·9	52	−1·5±0·4	Y	(4)
170	1987	4	28	E	2 446 913·566	Schaefer†	19	V	F	38·9	−77·0	100	50	50	9	11·6	11·5	3·0	23·0	40	0·3±0·2	Y	(1)
171	1987	4	28	E	2 446 913·566	Doggett	19	I		38·9	−77·1	100	50	50	9	11·6	11·5	3·0	23·0	40	0·3±0·2	N	(−1)
172	1987	4	28	E	2 446 913·566	Seidelman	19	I	B	38·9	−77·1	100	50	50	9	11·6	11·5	3·0	23·0	40	0·6±0·2	N	(−2)
173	1987	4	28	E	2 446 913·566	Slowik	19	I		38·9	−77·1	100	50	50	9	11·6	11·5	3·0	23·0	40	0·3±0·2	N	(−1)
174	1987	4	28	E	2 446 913·566	Chester	19	V	F	38·9	−77·1	100	50	50	9	11·6	11·5	3·0	23·0	40	0·3±0·2	Y	(1)
175	1987	4	28	E	2 446 913·566	Schmidt†	19	V	F	38·9	−77·1	100	50	50	9	11·6	11·5	3·0	23·0	40	0·3±0·2	Y	(1)
176	1987	4	28	E	2 446 913·566	Caton	19	I		36·2	−81·7	4600	55	60	6	11·7	11·7	2·8	23·2	40	0·5±0·2	N	(−2)
177	1987	4	28	E	2 446 913·566	ASU	19	V	F	36·2	−81·7	4600	55	60	6	11·7	11·7	2·8	23·2	40	0·5±0·2	Y	(2)
178	1987	4	28	E	2 446 913·566	ASU†	19	V		36·2	−81·7	4600	55	60	6	11·7	11·7	2·8	23·2	40	0·5±0·2	Y	(2)
179	1987	4	28	E	2 446 913·566	McLeod	19	V	F	26·7	−81·8	0	65	75	6	11·6	11·5	−2·8	22·9	40	−0·2±0·5	N	(−0)
180	1987	4	28	E	2 446 913·566	Williams	19	I		28·0	−82·5	0	65	75	6	11·6	11·6	−2·7	23·0	40	−0·1±0·5	Y	(0)
181	1987	4	28	E	2 446 913·566	Seidelman	19	I		33·7	−84·4	1100	60	65	8	11·8	11·8	2·6	23·3	41	0·2±0·3	N	(−1)
182	1987	4	28	E	2 446 913·566	Victor†	19	V	B	42·7	−84·5	800	65	50	9	11·9	11·6	3·6	23·7	41	0·6±0·3	Y	(2)
183	1987	4	28	E	2 446 913·566		19	V	F	42·7	−84·5	800	65	50	9	11·9	11·6	3·6	23·7	41	0·3±0·3	Y	(1)
184	1987	4	28	E	2 446 913·566	Byrd	19	I		28·0	−87·4	200	55	70	6	11·8	11·8	−2·7	23·3	41	0·2±0·3	N	(−1)
185	1987	4	28	E	2 446 913·566	Pitluga	19	I		40·8	−87·7	800	65	50	8	12·0	11·8	3·3	23·8	41	0·4±0·2	N	(−2)
186	1987	4	28	E	2 446 913·566	Richardson	19	V		30·0	−90·1	0	65	75	6	11·9	11·9	−2·6	23·6	41	0·0±0·5	N	(−0)
187	1987	4	28	E	2 446 913·566	Fry	19	I		41·6	−93·7	800	60	60	6	12·2	11·9	3·5	24·3	42	0·5±0·2	N	(−2)
188	1987	4	28	E	2 446 913·566	Duncombe	19	V		30·3	−97·7	600	60	70	6	12·1	12·1	−2·6	24·1	42	0·3±0·3	Y	(1)
189	1987	4	28	E	2 446 913·566	†	19	V	F	30·6	−104·0	7900	25	75	6	12·3	12·3	−2·6	24·6	43	1·0±0·2	Y	(6)
190	1987	4	28	E	2 446 913·566	†	19	V		30·6	−104·0	7900	25	75	6	12·3	12·3	−2·6	24·6	43	1·0±0·2	Y	(6)
191	1987	4	28	E	2 446 913·566	Chamberlain	19	V		40·7	−111·9	4300	50	60	6	12·7	12·5	3·6	25·5	44	0·9±0·2	Y	(5)
192	1987	4	28	E	2 446 913·566	Klemola	19	V		37·0	−122·0	4900	65	60	6	13·0	12·9	3·1	26·0	46	0·9±0·2	Y	(4)
193	1987	5	28	E	2 446 943·135	Schaefer	17	V		39·2	−105·5	6500	40	45	6	17·2	16·0	7·0	36·0	70	2·2±0·1	Y	( )
194	1987	6	25	M	2 446 972·734	Schaefer†	17	I	B	−30·1	−71·0	9100	30	40	4	9·8	4·5	−9·1	−18·0	38	−1·0±0·2	Y	(6)
195	1987	6	26	E	2 446 972·734	Schaefer†	17	I	B	−30·1	−71·0	9100	35	45	5	9·0	4·1	−8·5	16·4	35	−1·5±0·2	Y	(8)
196	1987	6	26	E	2 446 972·734	Victor	20	V	B	42·7	−84·5	800	60	75	9	10·4	9·5	5·0	20·2	42	−0·8±0·5	N	(−1)
197	1987	6	26	E	2 446 972·734		20	I		30·0	−100·0	1500	30	90	6	10·6	10·4	3·2	20·6	42	−0·4±0·4	Y	(1)
198	1987	6	26	E	2 446 972·734		20	I		39·8	−105·0	5500	50	70	8	10·9	10·0	5·0	21·5	44	−0·3±0·3	Y	(1)
199	1987	6	26	E	2 446 972·734	Chamberlain	20	V	B	40·7	−111·9	4300	35	75	6	11·1	10·0	5·4	22·0	45	0·3±0·2	Y	(1)
200	1987	6	26	E	2 446 972·734		20	V		33·5	−112·1	1080	20	95	6	11·0	10·5	4·0	21·6	44	−0·2±0·3	N	(−0)
201	1987	6	26	E	2 446 972·734	Klemola	20	V		37·0	−122·0	4900	70	60	6	11·3	10·5	4·8	22·5	46	0·1±0·3	Y	(0)

* References in col. 8: 1, Fotheringham 1910; 2, Fotheringham 1921; 3, Maunder 1911; 4, Bougon 1911; 5, MacKenzie 1921; 6, MacKenzie 1922; 7, Ashbrook 1972; 8, Ashbrook 1971; 9, McMahon 1972; 10, Hanford 1973; 11, Austin 1973; 12, Ashbrook 1977; 13, Patterson 1977; 14, Victor & Bakich 1977; 15, Ashbrook 1978a; 16, Ashbrook 1978b; 17, private communication from the observer; 18, Ashbrook 1979; 19, Doggett & Seidelmann 1988; 20, Doggett 1987.

† Indicates the presence of additional observers.

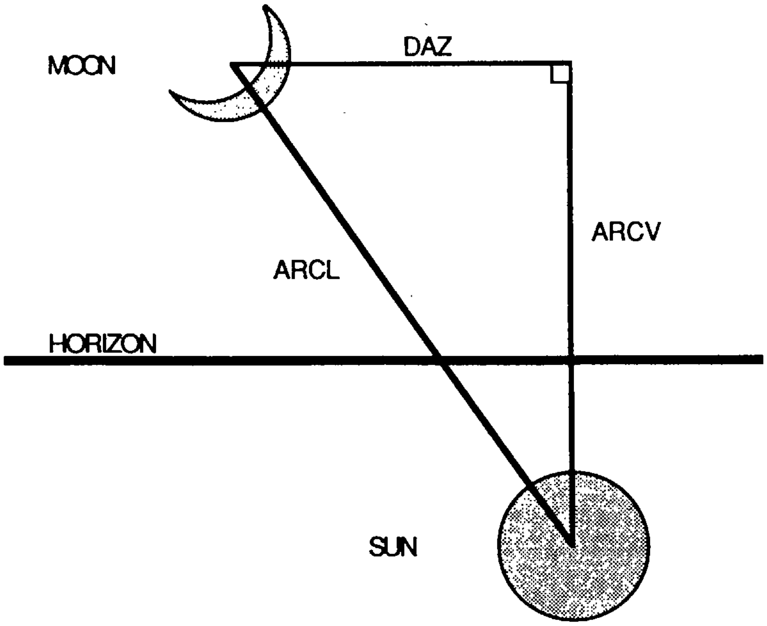

FIG. 1. Relative placement of the Sun and Moon. The three quantities ARCL, ARCV, and DAZ define a right spherical triangle with the hypotenuse connecting the geocentric centres of the Sun and Moon and the legs orthogonal to the altitude and azimuth coordinate system. No refraction or parallax corrections are included in the tabulated values, although these corrections are of course included in the calculation of lunar visibility of Schaefer's model. The tabulated quantities in Table I were calculated for the time of best visibility on the night of observation (unlike the discussions by Fotheringham, Maunder, and Ilyas) so that latitude effects will not be present.

Moon (in degrees with no parallax or refraction corrections). The relative azimuth (in degrees) is that of the centre of the Moon relative to the centre of the Sun, with a negative sign indicating that the Moon is more northerly along the horizon. The arc of light (ARCL), the arc of vision (ARCV), and the relative azimuth (DAZ) are illustrated in Fig. 1. The age of the Moon is given in hours of time from the instant of conjunction with negative values indicating observations in morning twilight. The moonset lagtime (column 21) is the length of time in minutes between the time of sunset (or moonrise) and moonset (or sunrise). Column 22 gives the visibility of the Moon in terms of its *R* value (see next paragraph) at the time of best visibility. The value in the column after the ' ± ' sign is the calculated one sigma error in the *R* value. Column 24 gives either a 'Y' or an 'N' depending on whether the model calculation of Schaefer (1988) agrees with the observation or not. The final column (25) gives the significance of the observation as calculated by this model, that is, *R* (column 22) divided by *DR* (column 23) rounded off to the nearest integer. A negative value indicates disagreement between model and observation. If no number is given in parentheses, then the observation is 'trivial' in the sense that the Moon was very easy to detect (i.e. the value is ten or greater).

The *R* value calculated from my model is a logarithmic measure of the visibility of the Moon. It is calculated as the log of the actual total brightness of the Moon divided by the total brightness of the Moon needed for visibility for the given observing conditions. A positive value implies that the Moon should be visible, while a negative value implies invisibility. So column 24 should be a 'Y' if the Moon is predicted to be visible (column 22 contains a positive number) and the Moon was actually observed (column 9 is a 'V'). Similarly, column 24 should be a 'Y' if column 22 is negative and column 9 is an 'I'. *R* is calculated such that an adult with average eyesight will have a 50 per cent probability of detecting the Moon in a cloudless sky for the given observing conditions for an *R* value of zero. My experience indicates that observing the Moon is difficult if *R* is less than 1 and is easy for *R* values greater than 2. The estimated error in *R* (DR in column 23) is calculated from the change in *R* when the observing conditions and the observer's detection probability are changed by 1 σ.

DISCUSSION

The data presented in the table can be used to evaluate the various prediction algorithms: the ancient Babylonians suggested a visibility criterion based on the time difference between sunset and moonset (Ilyas 1984). Their criterion claims that the Moon will be visible if this moonset timelag is greater than 48 minutes. Ilyas (1984) has demonstrated that this criterion should actually be stated as a weak function of season and latitude. The moonset lagtime criterion can be tested by comparing the lagtimes from column 21 of Table I with the visibilities given in column 9. In fact, over half of the positive sightings and over a quarter of the negative sightings are in contradiction with either the Babylonian rule or Ilyas' modification. There is a large selection effect in that (especially positive) observations tend to be reported if they are critical and hence with a greater chance of violating the criterion. The smallest lagtime for a visible Moon is 22 minutes (observation 44) while the largest lagtime for an invisible Moon is 84 minutes (observation 22), hence the uncertainty in the critical lagtime is at least 62 minutes. On average, the Moon will set 54 minutes later on any two successive days, so an uncertainty of over 54 minutes in the critical lagtime implies that no location on Earth can have a certain prediction by the moonset lagtime criterion.

The Moon's age has also been used as a criterion for predicting lunar visibility, typically with a cut off of around 24 hours for temperate latitudes. Ilyas (1983) has shown that the age is not a reliable criterion for temperate latitudes. His range of critical ages varies between 17 and 33 hours at a latitude of 30 degrees. However, even on the equator, the range of critical ages will vary from 16 to 25 hours. This range will increase when either the latitude is increased or the uncertainties of the weather are included. In the data from column 20 of Table I, the youngest visible Moon is 14·9 hours (observation 119) while the oldest invisible crescent is 51 hours (observation 22). Over half of the positive observations and a quarter of the negative observations violate the simple 24-hour criterion. Once again selection effects will weaken the force of this statistical argument. For every hour of uncertainty in the critical age, there will be additional 15 degrees in the

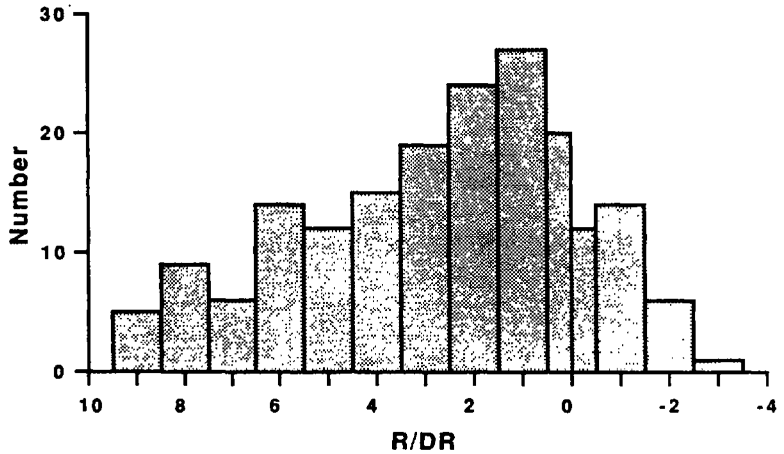

FIG. 2. Comparison of predictions and observations for Schaefer's model. This histogram shows the number of observations as a function of the calculated significance of detection. Along the horizontal axis, the significance (i.e. R divided by DR) is a measure of agreement between the observations and the model of Schaefer (1988). The values plotted are taken from the last column of Table I. Positive values indicate agreement between the model and observation, whereas negative values indicate disagreement. A total of 33 observations are not included in this histogram because they are 'trivial' in the sense that the R/DR is greater than 10. The rise in numbers as the ratio of R to DR is decreased is due to the selection effect that critical observations are preferentially reported. The fall to zero in the numbers to the right of zero is due to the lack of observations which greatly disagree with the model.

longitudinal extent of the zone of uncertainty. So even in the ideal case of the equator (with no weather variations) the zone of uncertainty will extend over 135 degrees of longitude. For realistic cases at temperate latitudes, the entire world will have an uncertain prediction.

The altitude/azimuth criterion of Fotheringham and others can be tested by plots of columns 9, 18, and 19 in Table I. The error in the critical altitude at sunset is over 2·5 degrees (compare observation 44 with 59, 171–3, 176, 180–1, 184–5, and 187 for small relative azimuth and observation 2 with 28, 99, and 100 for large relative azimuths). This is a lower limit on the practical range of uncertainty because there is a strong selection effect for the reporting sites to be of good quality and hence relatively uniform (whereas sites needing lunar visibility predictions can have a wide range of quality). Along a given latitude, the altitude at sunset will change by roughly half a degree times the cosine of the latitude for every extra hour of time, which adds an extra 15 degrees of longitude to the width of the zone of uncertainty. For temperate latitudes, this implies a zone of uncertainty greater than 105 degrees in longitude. For the specific case of Ilyas' implementation of Fotheringham's criterion (Ilyas 1984), observations 184, 185, and 187 are 40, 48, and 54 degrees away from the predicted line of critical visibility, implying the total width of the zone of uncertainty to be 108 degrees in longitude.

The R criterion of Schaefer (1988) can be tested by examining a histogram of the significances of the observation (the final column in Table 1) as presented in Fig. 2. Theoretically, I would predict that the histogram should rise as R is decreased to zero because of the selection effect to report only critical observations. For negative values of R, I would expect that the histogram should resemble a Gaussian distribution with a mean of zero and a variance of unity. The actual observed histogram apparently has the break at more like 0·5 and a standard deviation of 1·5. This implies that my model is slightly optimistic in its predictions and that the probabilities of error are slightly underestimated. Based on my experience with heliacal rise observations (Schaefer 1987), the relative excess of discrepant negative observations is undoubtedly due to unrecognized low-altitude clouds and haze layers. This effect is hard to quantify and has not been included in my model. However, since this is apparently a real if small effect, the probability distribution within the zone of uncertainty is slightly more pessimistic than my *a priori* calculations would predict. The size of the zone of uncertainty will vary widely from lunation to lunation. Typically, the zone will be largest during local summer because the atmospheric content of precipitable water is usually large and relatively unpredictable. Even for a given lunation, the zone will have a variable width as a function of latitude depending on how the climate varies. However, as an average, the width can be calculated by comparing the average change in R over one day with the largest uncertainty in R. The average change in R between the day of first visibility and the previous day is 4·3. The average value of DR (column 23 of Table I) is 0·23. The observation with the largest discrepancy (column 25) is 2·5 (observation 44), hence the average largest DR will be 0·57. This is 13 per cent of the daily variation. The full width of the zone of uncertainty will be on average 13 per cent of 360 degrees, or 47 degrees in extent.

CONCLUSIONS

I have collected 201 observations of lunar visibility and have evaluated several prediction algorithms. Criteria involving the moonset lagtime and the Moon's age are found to have zones of uncertainty which typically cover the entire Earth. The altitude/azimuth criterion is found to have a zone of uncertainty with a width over 105 degrees in longitude. The algorithm of Schaefer (1988) is found to have a zone of uncertainty which is less than half the size of any other algorithm.

REFERENCES

Abbott, C.G. *et al.*, 1908–54. *Ann. astrophys. Obs. Smithson. Instn*, vols 2–7.
Angione, R.J. & de Vaucouleurs, G., 1986. *Publs astr. Soc. Pacif.*, **98**, 1201.
Ashbrook, J., 1971. *Sky Telesc.*, **42**, 78.
Ashbrook, J., 1972. *Sky Telesc.*, **43**, 95.
Ashbrook, J., 1977. *Sky Telesc.*, **53**, 440.
Ashbrook, J., 1978a. *Sky Telesc.*, **55**, 358.
Ashbrook, J., 1978b. *Sky Telesc.*, **55**, 453.
Ashbrook, J., 1979. *Sky Telesc.*, **57**, 403.
Austin, R.R.D., 1973. *Sky Telesc.*, **46**, 139.
Bougon, J., 1911. *J. Br. astr. Ass.*, **22**, 48.
Bruin, F., 1977. *Vistas in Astron.*, **21**, 331.
Coleman, A., 1932. *J. Br. astr. Ass.*, **42**, 219.
Doggett, L. & Seidelmann, P.K., 1988. in *Proceedings of the I.I.I.T. Conference on the Lunar Calendar*, ed. I. Ahmad (in the press).
Flowers, E.C., McCormick, R.A. & Kurfis, K.R., 1969. *J. Appl. Meteor.*, **8**, 955.
Fotheringham, J.K., 1910. *Mon. Not. R. astr. Soc.*, **70**, 527.

Fotheringham, J.K., 1921. *Observatory*, **44**, 308.
Gutierrez-Moreno, A., Moreno, H. & Cortes, G., 1982. *Publs astr. Soc. Pacif.*, **94**, 722.
Hanford, J., 1973. *Sky Telesc.*, **45**, 403.
Ilyas, M., 1981. *Q. Jl R. astr. Soc.*, **22**, 154.
Ilyas, M., 1983. *Observatory*, **103**, 26.
Ilyas, M., 1984. *Islamic Calendar*, *Times* & *Qibla* (Kuala Lumpur, Berita).
Lockwood, G.W. & Thompson, D.T., 1986. *Astron. J.*, **92**, 976.
MacKenzie, T., 1921. *J. Br. astr. Ass.*, **31**, 380.
MacKenzie, T., 1922. *J. Br. astr. Ass.*, **32**, 311.
Maunder, E.W., 1911. *J. Br. astr. Ass.*, **21**, 355.
McMahon, J.H., 1972. *Sky Telesc.*, **43**, 295.
Mendonca, B.G., Hanson, K.J. & DeLuisi, J.J., 1978. *Science*, **202**, 513.
Miles, R., 1983. *Int. Am.-Prof. Photoelectric Phot. Comm.* no. 13, 12.
Morrison, B.L., 1966. *U.S. Naval Obs. Circ.* no. 112.
Patterson, K.A., 1977. *Sky Telesc.*, **53**, 174.
Pearce, E.A. & Smith, C.G., 1984. *World Weather Guide* (New York Times).
Przybylski, A., 1964. *Acta Astron.*, **14**, 285.
Rufener, F., 1986. *ESO Mess.*, no. 44, 32.
Schaefer, B.E., 1987. *Archaeoastron.* (J.H.A. xviii), no. 11, S19.
Schaefer, B.E., 1988. in *Proceedings of the I.I.I.T. Conference on the Lunar Calendar*, ed. I. Ahmad (in the press).
Victor, R.C. & Bakich, M., 1977. *Sky Telesc.*, **53**, 91.