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The Length of the Lunar Crescent

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SUMMARY

Observation of the young Moon has from time to time suggested that the outer terminator of the illuminated crescent is less than the theoretical value of 180° often by a substantial margin. This phenomena is not observed at every new Moon. It is shown that deformations of the Moon's figure are insufficiently large to account for the observed effect but it is shown that the effects of seeing in the Earth's atmosphere account for the observations both qualitatively and quantitatively.

1 INTRODUCTION

In all elementary treatments of lunar phase, the figure of the Moon is assumed to be that of a sphere. In such circumstances, the shape of the Moon as seen from the Earth will be bounded by a circle. The outer terminator of the lunar crescent will always be a full half-circle (except at the instant of full Moon) while the inner terminator will be a full half-ellipse whose major axis is also the diameter of the outer terminator. However, it is observed (e.g. Danjon 1932, 1936 and Ashbrook 1971, 1972 for a recent rediscussion) that the outer terminator of the Moon need not be a full half-circle and can indeed be considerably less than this.

Danjon sought an explanation in terms of shadowing by lunar mountainous terrain. An alternative might also lie in departures from sphericity in the lunar figure. Modern measurements have shown that variations in height of terrain and departures from true sphericity are less than 0·6 per cent of the lunar radius. This means the figure of the Moon, from the point of view of lunar phase, may be assumed to be so closely spherical, that the elementary assumption of sphericity is sufficient. It will be shown in Section 4 of this paper that gross departures from sphericity are required to give an effect of the order observed.

If physical properties of the Moon, e.g. variations in elevation of terrain, distortions of figure, are not responsible for the observed phenomena, one can ask if effects of inclination of the Moon's orbit to the ecliptic or libration can supply an explanation. Clearly these properties can also be dismissed since they can in no way affect the geometry of shadow production on a spherical Moon.

A further alternative is to contest the observations. Severe shortening of the outer lunar terminator is only claimed by Danjon for observation of the extremely young Moon when optical aid is needed in order to observe the lunar crescent. At times when naked eye observation would be feasible the angular shortening would be of the order of 60°. This is still striking but is not necessarily noteworthy by the casual observer. The effect is not seen at every lunation so that doubt has been cast upon the validity of Danjon's observations. However, the beautiful photography of McClure (see Fig. 1) and the photographs reproduced by Danjon show that the phenomenon is real enough.

An interesting side issue was raised by Danjon's work. If, as Danjon claimed, the shortening of the outer terminator was a shadowing effect by lunar mountains, there was the clear possibility that it would be impossible to see the Moon at all until the elongation of the Moon from the Sun has reached a value greater than 7°. The same possibility would also exist if the phenomenon were a result of distortion of the Moon's figure. This would mean that there is a period spanning the instant of new Moon where the Moon would be invisible to terrestrial observers. It might be that such an *observation* might be of value in regulating lunar calendars. For example, use of this observation might not be in conflict with the Koran ('we do not write, nor do we reckon', Bruin 1977) for the regulation of the Muslim lunar calendar. The beginning of the month could clearly be fixed when the elongation of the Moon from the Sun was 7°.

If, as will be shown in this paper, the observed phenomena is real but not an intrinsic property of the Moon, the value of the observation for calendrical regulation is diminished. It will be shown that atmospheric seeing provides an acceptable account of the observations. Atmospheric seeing arises through the unsteadiness of the Earth's atmosphere and is a phenomenon well appreciated by astronomers. Usually, its effects are diminished resolution and great pains, energy and expense are dissipated in endeavouring to select sites for observatories where atmospheric seeing is a minimum. The atmosphere undergoes changes of density and temperature over short time-scales (by comparison with the length of time for an astronomical observation). These changes produce changes in the refractive index of the atmosphere and result in movement of images in the focal plane of telescopes. Instead of a compact image, the image is smeared out over an area determined by the changes of atmospheric refractive index. The size of the image is determined in seconds of arc and this size is used as measure of the seeing. For example if the image size is 2 arcsec then the seeing is said to be 2 arcsec seeing and so on. It is this same phenomenon which we suggest affects the visibility of the youthful lunar crescent. Briefly, if the lunar crescent is thinner than the size of the seeing, the crescent will be spread to the size of the seeing. This means that the illumination of the crescent will be spread over a wider area and the illumination for unit area thereby reduced. The contrast between crescent and sky having been reduced by the seeing therefore makes the thinner parts of the crescent more difficult to detect. This is as true of naked eye observation as with telescopic detection. It will be shown in Section 5 that seeing effects explain both the trend of the observations and the size of the effect. Since seeing is variable from time to time at a single site and variable from site to site, then this phenomenon provides little basis for calendrical regulation. As might be expected, the phenomenon being a consequence of the properties of the Earth's atmosphere, the young crescent can be seen from space very much earlier than by terrestrial observers.

2 THE OBSERVATIONS

The largest single body of data on the variation of the length of the outer terminator of the Moon was given by Danjon (1936). Danjon (1932) had previously discussed this phenomenon giving a single illustration and the theory of the measurements that he had made. His 1936 paper gave a further set of illustrations and reported observations from several other observers. Danjon took a mean of these observations to give the variation in the length of the outer terminator in terms of the elongation (or, more accurately, 180° – elongation) of the Earth from the Sun as viewed from the centre of the Moon. These mean values are given in the final column of Table II. These results show that the effect is most severe close to the instant of new Moon, the severity of the shortening decreasing rapidly as the Moon waxes. The effect is shown very well by the beautiful photograph of McClure (1971) which appeared in *Sky and Telescope* and is reproduced here as Fig. 1.

Danjon (1932) argued that since the outer terminator of the Moon was shortened then the inner terminator must cut the great circle defining the hemisphere of the Moon visible from the Earth at two points which were not connectable by a diameter of the great circle but rather by some other chord. He envisaged the situation of Fig. 2. As we shall show later, this geometrical picture is incorrect. Danjon might have considered the spherical triangle QBZ where Q is a cusp of the terminator, QZE is the lunar meridian from

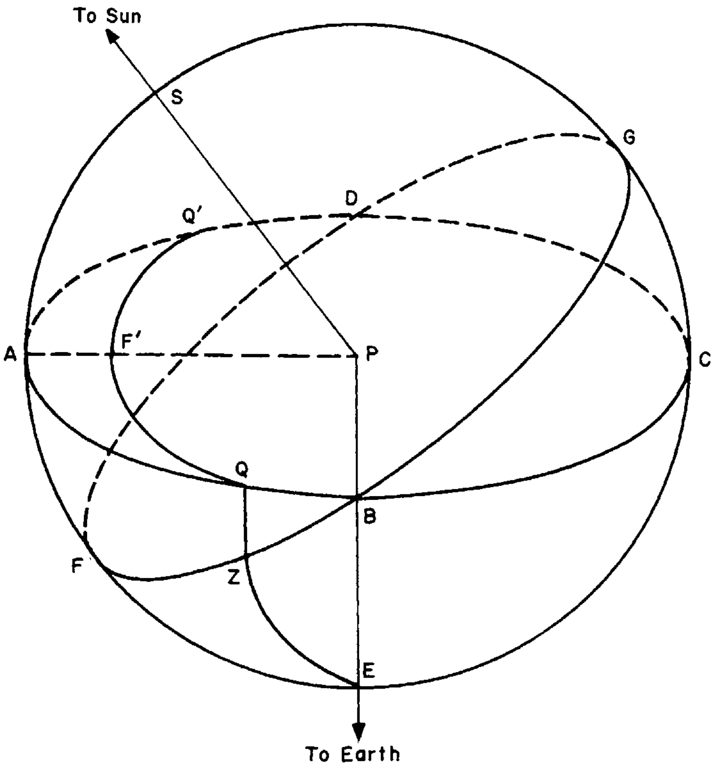

FIG. 2. Danjon's geometry for shortening the outer terminator.

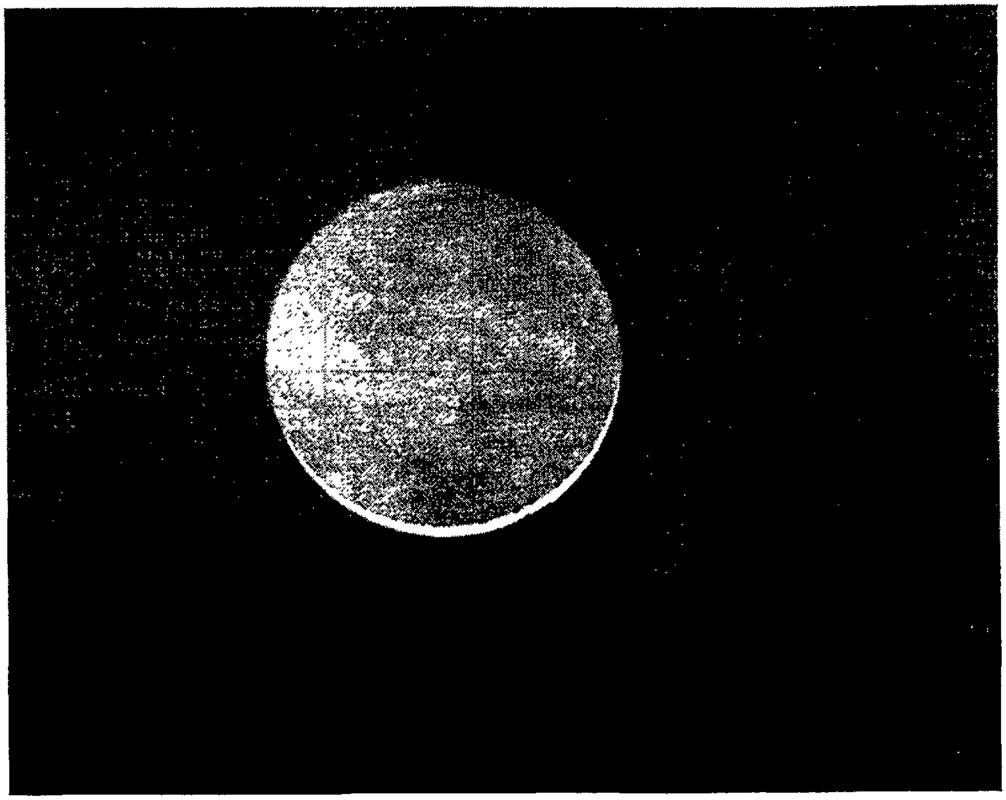

FIG. 1. The shortened outer terminator. This photograph was obtained by Mr A. McClure of Los Angeles and is reproduced by permission of Mr McClure and *Sky and Telescope*. The photograph was first published as the cover picture of *Sky and Telescope* in 1961 March and was reproduced again in *Sky and Telescope* in 1971, associated with the article on this phenomenon by J. Ashbrook.

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the point E, where the Earth–Moon line of centres cuts the Moon's surface, to the great circle ABCD defining the visible portion of the Moon. FZBD is the hemisphere which should be illuminated by the Sun. Because of topographical shadowing Danjon assumed that the illuminated area was reduced. Danjon called the arc QZ (denoted by α) the deficiency arc. The angle $A\hat{B}F$ he denoted by a ($\equiv \theta$ later in this paper); a is therefore $180° - \chi$ where $\chi = S\hat{P}E$ is the elongation of the Earth from the Sun as viewed from the centre of the Moon: $QB = 90° - \omega$ where ω is the semi-length of the outer terminator. Danjon's values for 2ω are tabulated in the final column of Table III. Application of the 4-parts rule of spherical trigonometry (see McNally 1974) to the spherical triangle QBZ gives

$$\tan \alpha = \tan a \cos \omega. \tag{2.1}$$

Danjon, in fact, found

$$\sin \alpha = \sin a \cos \omega \tag{2.2}$$

which could have been the result of his using a plane geometrical argument, through an assumption that the plane defining the illuminated hemisphere was not FZBD but a plane parallel to this plane though Q, Q′. The numerical difference between equations (2.1) and (2.2) is not significant since α is a small angle ($\lesssim 7°$); even though a can become large ω tends to 90° rapidly, e.g. $\omega = 87°$ by the time $a = 30°$.

Given the photographic evidence for considerable shortening of the outer terminator of the Moon there can be no doubt about the reality of the phenomenon. The problem is therefore to reconcile expectation with observation. A résumé of the theory of lunar phase is given in Section 3, while it is shown that the distortion of figure required to produce the effect is too gross to be supported by observation in Section 4; Section 5 is devoted to showing that the observations are matched by supposing the phenomenon is produced by seeing in the Earth's atmosphere.

3 LUNAR PHASE

The figure of the Moon has been shown to closely approximate to a sphere. Cook (1980), quoting Sjogren & Wollenhaupt (1976), gives the values tabulated in Table I for the lengths of the principal axes *a, b, c* of the Moon's figure.

TABLE I

Sjogren & Wollenhaupt's values for length of the principal axes of the Moon's figure

	Maria (km)	Highlands (km)
a	1736·6	1738·1
b	1735·0	1738·2
c	1733·0	1738·0

There is a maximum variation of 5·2 km from the shortest to the longest estimate, a variation of 0·3 per cent. The work of Kuala *et al.* (1973) seems to suggest that local variations of height across the Moon are contained within the range ± 5 km. Other lunar properties, e.g. Brown *et al.* (1974) suggest variation in the range ± 1 km. Taking the maximum departure to be 10 km the departure from true sphericity is only 0·6 per cent. From the point of view of the determination of lunar phase such departures are too insignificant to alter the assumption that the Moon may be regarded as spherical.

Most elementary books, e.g. *Positional Astronomy* by D. McNally (Muller 1974), on positional astronomy give the theory of lunar phase. The geometrical situation is illustrated in Fig. 3 in which P is the centre of the Moon, PE is the direction towards the Earth and PS is the direction towards the Sun. The great circle ABCD defines the limits of the visible lunar hemisphere while the great circle FBGD defines the limits of the illuminated lunar hemisphere. The outer terminator is the semi-circle BAD and the inner terminator is the semi-ellipse BF′D. It is clear that the outer terminator must be a semi-circle provided the Moon is spherical in shape, i.e. we may measure the outer terminator to have an angular length of 180°. The fact that the plane of the Moon's orbit does not lie in the ecliptic, or that the Moon librates does not alter the consequences of this geometry. If Fig. 3 is interpreted simply in terms of directions, the only thing which determines the length of the outer terminator is the shape of the Moon.

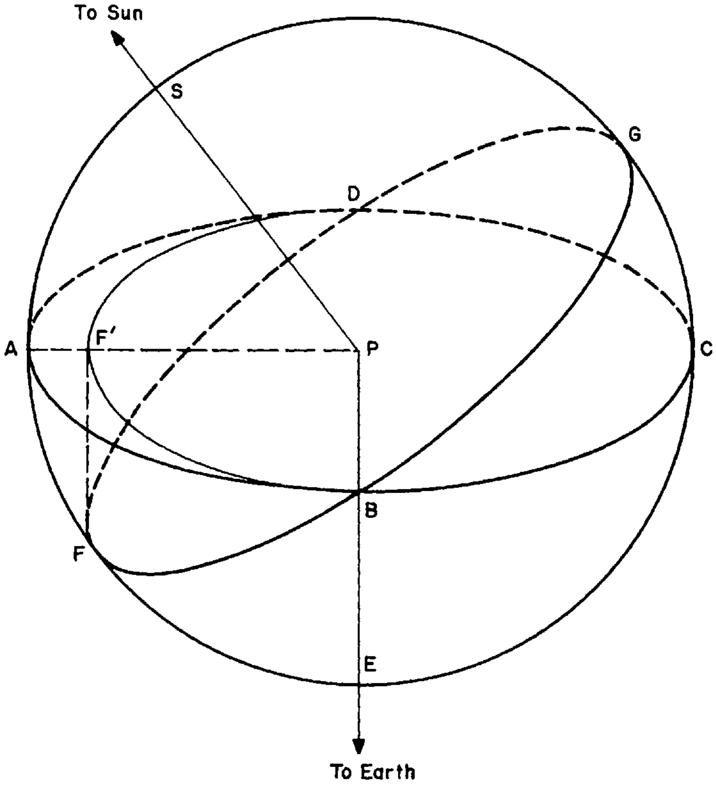

FIG. 3. The geometrical situation for the determination of lunar phase. Reproduced from *Positional Astronomy* by D.McNally (1974).

The shape of the illuminated part of the Moon's disc is determined by projecting the boundary DFB of the illuminated part of visible lunar hemisphere into the plane ABCD. The semi-circle DFB projects into the semi-ellipse DF′B. The major axis of this ellipse is BPD of length $2R$ where R is the mean lunar radius. The length of the semi-minor axis of this ellipse is $PF' = R \cos \theta$ where $\theta = 180° - \chi$, χ being the elongation of the Earth from

Sun as viewed from the Moon's centre, i.e. $S\hat{P}E$. The illuminated area A of the Moon is thus

$$A = \tfrac{1}{2}\pi R^2 - \tfrac{1}{2}\pi R\cdot \mathrm{PF}' = \tfrac{1}{2}\pi R^2\,\{1 - \cos\theta\} \qquad (3.1)$$

and the phase is defined to be

$$\frac{A}{\pi R^2} = \tfrac{1}{2}(1 - \cos\theta) = \tfrac{1}{2}(1 + \mathrm{cost}\,\chi). \qquad (3.2)$$

The method of determining χ (or θ) when the orbits of Earth and Moon are not coplanar may be found from the Explanatory Supplement to the *Astronomical Ephemeris* (HMSO, 1961, p.311). If the assumption of coplanarity is assumed $\chi = 180°$ at new Moon, $= 270°, 90°$ at quadrature, $= 0°$ at full Moon.

4 THE EFFECT OF DISTORTING THE SHAPE OF THE MOON

As was pointed out above, a Moon of spherical shape must be observed, when young or old, with an outer terminator which is a semi-circle. Distortion of the shape of the Moon could alter the geometry of the shadow. Distortion of the Moon's shape for the hemisphere facing the Earth is all that is required. Suppose, for the sake of finding the order of magnitude of the necessary distortion, that the side of the Moon facing the Earth is a spheroid of revolution. The Moon will therefore still appear circular but the half-meridian normal to the Earth–Moon line of centres will have an elliptical profile as illustrated in Fig. 4.

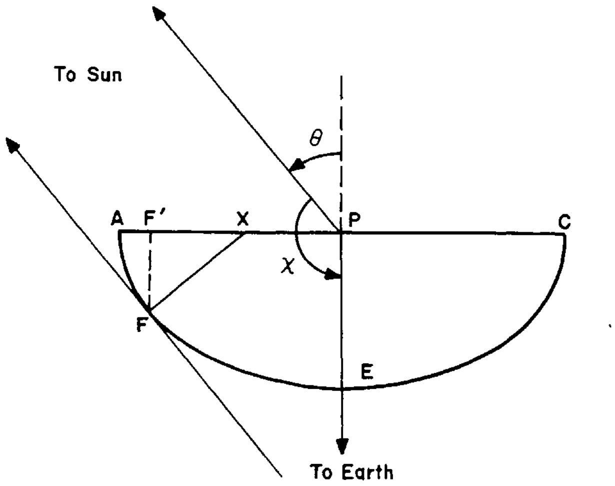

FIG. 4. An elliptical distortion of the Moon.

In Fig. 4 the limitation of illumination (as in the case of a spherical Moon) is provided by the ray of sunlight tangential to the lunar surface at F. The boundary of the shadow now does not define a plane which includes the centre of the Moon. That plane cuts the great circle ABCD (of Fig. 2) at two points Q and Q′. Y is the midpoint of QQ′. The lunar crescent is then formed as shown in Fig. 5. A, B, C, D, P are as in Figs 2 and 3. Clearly the length of the outer terminator is now reduced. Denote the angular half-

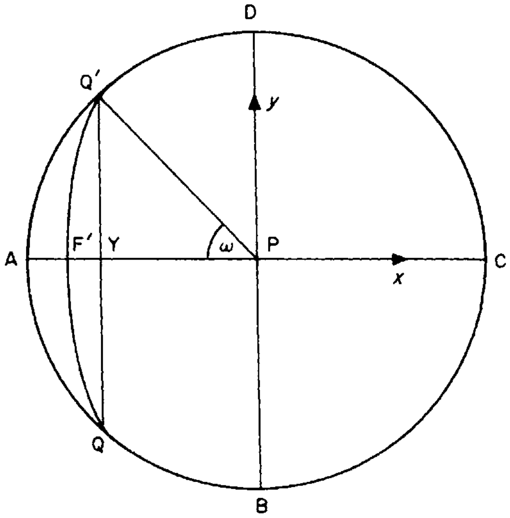

FIG. 5. The crescent for a distorted Moon.

length of the outer terminator by $\omega = A\hat{P}Q'$. Danjon claims that the shortest terminator observed gives $\omega = 33°$ ($2\omega = 66°$) when $\theta = 8°$. Hence, since

$$YP = p = R\cos\omega,\ p/R = 0{\cdot}84 \text{ for } \omega = 33°. \tag{4.1}$$

Using Fig. 4 it is clear that the slope of the tangent to the ellipse at F is $-\cot 8° = -7{\cdot}1$. Suppose that the figure of the Moon is represented by an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{4.2}$$

where a and b are the lengths of its semi-major and semi-minor axes respectively. From equation (4.2) the slope of the tangent to the ellipse at any point x, y is

$$\frac{dy}{dx} = -\frac{x}{y}\frac{b^2}{a^2}. \tag{4.3}$$

The point x, y is also the point of intersection of the normal with the ellipse. The equation of the normal is

$$y = mx + c, \tag{4.4}$$

where $m = \tan 8°$ and c is determined since from Fig. 5 $y = 0$ for $x = p$. Expressing all quantities in units of R the radius of the Moon ($a \equiv R$), $c = -0{\cdot}12$ and $m = 0{\cdot}14$ ($= \tan 8°$). The x coordinate of F′ must lie in the range $1{\cdot}0 > x > 0{\cdot}84$ since x must exceed p and so the corresponding value of y for F must lie in the range $0 < y < 0{\cdot}023$. The value of x/y must therefore exceed 37. Hence $\frac{b^2}{a^2}$ must be less than $0{\cdot}0019$ or $\frac{b}{a} < 0{\cdot}044$.

The flattening implied (i.e. the greatest extent of the Moon towards the Earth is 762 km from the centre) is so excessive that it is wholly unreasonable to suppose that the shortening of the outer terminator just after new Moon can be caused by distortion of the Moon's figure.

A similar argument can be applied when the shortening of the outer terminator is small say < 5°. It is still clear that too gross a distortion to be allowed by the observed lunar parameters is required. Furthermore distortion of the lunar figure would give a pattern of observation which repeats with each lunation. The observations therefore simply rule out an explanation based on distortion of the lunar figure.

5 EFFECT OF THE EARTH'S ATMOSPHERE

Having ruled out distortion of the Moon as a cause of the observed shortening of the outer terminator, the list of possible lunar properties giving rise to the phenomenon is now exhausted. Danjon's geometry of Fig. 2 is incorrect since the Moon is clearly spherical in shape. The geometry of Fig. 3 must apply. The only remaining way the phenomenon can be explained is through means of observation. The item which is common to all observations is that observation is through the Earth's atmosphere. The young Moon is usually first observed by the naked eye when close to the horizon just after sunset. Such circumstances are ideal for maximizing instability in the atmosphere and ensuring maximum seeing. For many sites, such as London, seeing as great as 5 arcsec is not uncommon and twice this value would not be at all untoward near the horizon at sunset. Given seeing of this order the illuminated lunar crescent would suffer expansion to the order of the seeing. The brightness would drop since the same total illumination is now spread over a wider area and so the thin parts of the crescent near the cusp would be harder to see against a bright sky background. For 5 arcsec seeing there would be little point in expecting to see those parts of the lunar crescent with a width less than this value. This is not a prediction in the sense that if the width of the crescent is 5 arcsec then it must be seen and if the width is less than 5 arcsec it cannot be seen. The detection of illuminated narrow lines is often achieved even when the width of the line is less than the theoretical limit of resolution. However, the smearing of a thin crescent by atmospheric seeing will cause the brightness of the crescent to drop and a small decrease in brightness is sufficient to render the crescent invisible in contrast with a brighter sky background. The younger the Moon, the brighter will be the sky background and so the more sensitive will observation be to the effect of the seeing. We will use the size of the seeing as a bench mark to find out when seeing is likely to be significant and when not. We shall find that the effects of seeing clearly model the mean observed results of Danjon (1936). Actual observation will depart from the theory given below since the actual length of outer terminator seen will be a function of altitude of observation, relative contrast in brightness between Moon and sky, the details of resolution of narrow bright lines as well as the seeing. Clearly as the moon waxes both the width of the crescent and the altitude of the Moon will increase so reducing the effect of seeing and will be seen in an increasingly darkening sky as the Moon draws away from the Sun. Hence the length of the outer terminator should increase rapidly to a full semi-circle as the Moon waxes. This is observed and is reproduced by theory below.

The effect can be demonstrated by means of a simple computation. The lunar crescent is illustrated in Fig. 6. The lettering in Fig. 6 has the same meaning as in Fig. 3 with the addition of points H, H′ on the outer and inner terminators respectively. Coordinate axes are chosen such at the x-axis lies along PD and the y-axis along PF′ A. Let PH have length R – the diameter of the Moon and PH′ have length r^*. Denote the angle $H\hat{P}D$ by ϕ. The quantity $HH' = R - r^* = \Delta R$ will be used as a convenient measure of the thickness of the crescent and can be compared with the size of the atmospheric seeing.

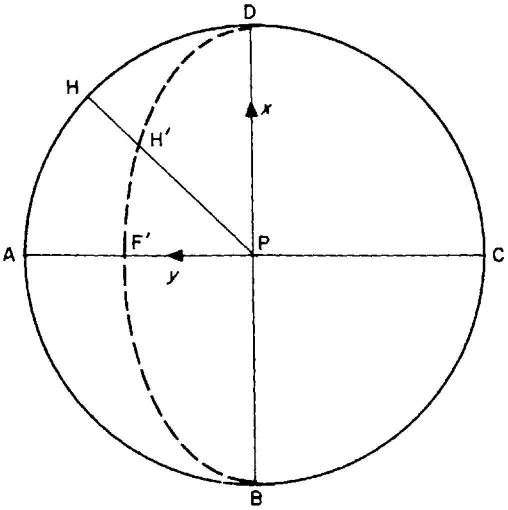

FIG. 6. The geometry of the lunar crescent.

The shape of the inner terminator of the crescent is a semi-ellipse whose semi-major axis is $PF' = R\cos\theta$ where $\theta = 180^\circ - \chi$, χ being the elongation of the Earth from the Sun as viewed from the Moon's centre P and whose semi-major axis is PD = PB = R the radius of the Moon. One may then write

$$\frac{x^2}{R^2} + \frac{y^2}{R^2\cos^2\theta} = 1 \tag{5.1}$$

for the equation of that ellipse. The line PH has equation

$$y = x\tan\phi. \tag{5.2}$$

Also

$$y = r^*\sin\phi. \tag{5.3}$$

Substitution for x from equation (5.2) in equation (5.1) gives

$$\frac{y^2}{R^2\tan^2\phi} + \frac{y^2}{R^2\cos^2\theta} = 1$$

or

$$y = \frac{R}{(\cot^2\phi + \sec^2\theta)^{1/2}}, \tag{5.4}$$

choosing the positive sign of the square root. Using equations (5.3) and (5.4)

$$r^* = \frac{R}{\sin\phi\,\{\cot^2\phi + \sec^2\theta\}^{1/2}} = \frac{R\cos\theta}{(\cos^2\phi\cos^2\theta + \sin^2\phi)^{1/2}} \qquad (5.5)$$

again taking the positive square root.
Then the width of the crescent is

$$\Delta R = R - r^* = R\left(1 - \frac{\cos\theta}{(\cos^2\phi\cos^2\theta + \sin^2\phi)^{1/2}}\right). \qquad (5.6)$$

Assuming a mean angular radius $R = 932$ arcsec for the Moon, ΔR (in seconds of arc) is tabulated as a function of θ, ϕ in Table II.

A small value of ϕ denotes a position on the outer terminator near the cusp while $\phi \sim 90°$ denotes a position near the centre of the outer terminator. It is clear from Table II that near new Moon the width of the illuminated crescent is very small near the cusp. Indeed, it is not until θ has reached 50° that the width of the crescent at 5° from the cusp exceeds 5 arcsec. Since seeing in many situations can commonly be of this order, the outer terminator will be readily observed shortened. If $\Delta R \simeq$ seeing then the length of the outer terminator initially unaffected by seeing can be determined since

$$\omega = 90° - \phi\ (\Delta R = \text{seeing}). \qquad (5.7)$$

Table II can be used in this way to locate ω. To better illustrate the situation the value of 2ω has been computed on the basis of a fixed value of the seeing $= \Delta R$. This determines the length of the terminator over which the width of the illuminated crescent exceeds that fixed value. In this way one has a measure of the length of the outer terminator unaffected by seeing. Equation (5.6) adjusts to give

TABLE II

ΔR (arcsec) as a function of θ, ϕ

ϕ θ	5	10	20	30	40	50	60	70	80	90
5	0·027	0·108	0·417	0·890	1·47	2·09	2·66	3·13	3·44	3·55
10	0·110	0·437	1·69	3·60	5·93	8·39	10·7	12·5	13·7	14·2
15	0·254	1·01	3·89	8·25	13·5	19·0	24·1	28·2	30·9	31·8
20	0·469	1·86	7·14	15·1	24·5	34·2	43·1	50·2	54·7	56·2
25	0·769	3·04	11·6	24·3	39·2	54·3	67·8	78·4	85·0	87·3
30	1·18	4·65	17·7	36·6	58·2	79·6	98·4	112	122	125
35	1·73	6·81	25·6	52·4	82·1	111	135	153	165	169
40	2·48	9·74	36·2	72·6	112	148	178	200	214	218
50	4·99	19·3	68·9	131	192	244	284	311	328	333
60	10·48	39·5	130	228	309	371	415	444	460	466
70	25·6	90·8	253	384	473	532	571	595	609	613
80	96·5	268	505	622	685	723	746	760	768	770
90	932	932	932	932	932	932	932	932	932	932

$$\cos\phi = \frac{1}{\sin\theta}\left\{1 - \frac{\cos^2\theta}{\left(1 - \frac{\Delta R}{R}\right)^2}\right\}^{1/2} \qquad (5.8)$$

so that 2ω can be determined from (5.7). Clearly this does not *predict* the length of the outer terminator since the actual length would be influenced by additional factors as well as the one under discussion. Values of 2ω determined using (5·7) are tabulated in Table III for various values of the 'seeing', i.e. ΔR and as a function of θ. For the purpose of comparison Danjon's (1936) mean values are included.

TABLE III

The length of the outer terminator versus 'seeing' (ΔR, arcsec) and θ

Seeing θ	2	4	6	8	Danjon
5	82·8	—	—	—	—
10	136	116	99·4	83·2	104
15	152	139	130	121	142
20	159	150	143	138	158
25	164	157	152	147	167
30	167	161	157	154	173·2
35	169	165	161	158	177·6
40	171	167	164	162	180
50	174	171	169	167	182·6
60	176	174	172	171	183·0
70	177	176	175	174	No inf.
80	179	178	178	177	No inf.
90	180	180	180	180	183·6

It is at once clear from Table III that for $\theta = 5^\circ$, seeing in excess of 4 arcsec will affect the whole illuminated crescent. This is why Danjon found that the Moon could not be seen from the ground for $\theta \leqslant 7^\circ$. It should be noticed that the Moon should be seen from space for $\theta \leqslant 7^\circ$. It is interesting to note that, while Danjon's mean results are compiled from observed values with reference to local seeing conditions, there is a trend to find lower values of seeing as the Moon ages. Interpretation of Danjon's results suggest a range of mean seeing of 6 arcsec at $\theta = 10^\circ$ to 2 arcsec at $\theta = 25^\circ$. This would be expected since the Moon is drawing away from the Sun and will be observed at progressively high altitudes. One would therefore expect to see such an improvement in the seeing.

It will also be noted in Table III that the trend of 2ω with θ, for the atmospheric seeing hypothesis is the same as Danjon's mean values. It would therefore seem that the phenomenon of an observed shortening of the length of the Moon's outer terminator is a straightforward consequence of atmospheric seeing.

However, Danjon's other observations that the terminator can lengthen to be greater than 180° does not receive an entirely happy explanation in terms of atmospheric seeing, though it could make some contribution in the right direction. For 2 arcsec seeing extension of the outer terminator by $0^\circ{\cdot}25$ and for 10 arcsec seeing extension by $1^\circ{\cdot}23$ would be predicted. The maximum extension observed by Danjon was $3^\circ{\cdot}6$. Clearly only in conditions of very poor seeing could seeing account for one-third of the observed extension. Treating the full $3^\circ{\cdot}6$ excess extension wholly as observational error, the percentage error is 2 per cent. We have shown that up to one-third of this could be a consequence of seeing. Measurement errors of the order of 1·5 per cent would not be unreasonable in this type of problem when a precise decision on where the terminator actually ends is not easy to make. This point is reinforced by reference to the illustration in Danjon's (1932) paper which shows a blob of illuminated surface beyond the end of the unbroken crescent. Therefore while atmospheric seeing seems to be a fair explanation for the observations, there is a clear implication that other factors may influence the appearance of the crescent over lengths less than 3°·6.

Nevertheless the effects of atmospheric seeing do allow close reproduction of the observations of the gross crescent and is clearly the major phenomenon affecting how the young crescent is observed from the Earth. It should be stressed again that the model used in this paper is not prescriptive of the length of the outer terminator which will be observed, but only an indication of the expected range for the length of the outer terminator. Other variables such as atmospheric clarity, contrast with sky background and visual response will all modify how the lunar crescent is actually observed by an individual or instrument. However, these latter modifications will be secondary to the dominant effect of atmospheric seeing.

6 VISIBILITY OF THE MOON

Given that atmospheric seeing causes the observed shortening, a new variable is introduced into the problem of early detection of the new Moon. Not only is the phase, contrast between lunar illumination and sky background, visual acuity of the observer important but so now is atmospheric seeing. Clearly one must now insist that in order to detect the new Moon early, conditions of good seeing must hold in order to maximize the length of the outer terminator. Observers used to good seeing will therefore find difficulty in believing that the Moon's outer terminator could diminish to a total length of 66° at $\theta = 8°$ as found by Danjon. Observers used to poor seeing will expect to see the young Moon with a shortened terminator.

These observations by Danjon give no basis for a method of fixing the beginning of a lunar month. Given that a particular site has a seasonal mean for atmospheric seeing, there is clearly little expectation of seeing the young Moon during the period when seeing effects are of the same order as the width of the crescent. However, even for sites of poor average seeing, conditions of good seeing will occasionally occur. There is always the chance that an exceptional combination of visual acuity, atmospheric clarity and stability will permit an earlier sighting of the new Moon than had previously been the case. Nevertheless, on average for sites where mean seeing is > 2 arcsec there will be a time near new Moon when it is quite unlikely that the new Moon will be seen from the ground even with optical aid. Danjon put the period as defined by $\theta = \pm 7°$ whereas this work would suggest $\theta = \pm 5°$ might be safer. Therefore, this phenomenon is not of significant value in the regulation of explicit lunar calendars.

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