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Limiting altitude separation in the new Moon's first visibility criterion

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Summary. The minimum altitude separation between the moon and the setting sun for new moon's earliest visibility is found to be close to 4°. This result together with an earlier study on limiting elongation makes the new Moon's earliest visibility criterion most complete and comprehensive.

Key words: celestial mechanics – planets and satellites: Moon

1. Introduction

New Moon's earliest visibility prediction has attracted astronomers' serious interest from very early times. Already, by 2000 B.C., Babylonians had developed a simple observational "Moonset lag" criterion which was nevertheless quite good at lower latitudes (Ilyas, 1986). It remained in effective calendrical use until the later parts of the medieval era even though its limitations had been recognized much earlier especially by the Muslims in the 9th Century A.D. e.g. by Al-Battani (see Bruin, 1977). The Hindus and more importantly Muslims studied the matter at length using physical considerations and developed the universal forms of the criterion (see Ilyas, 1984). However, it was not until the beginning of the 20th century that a modern form of the criterion was developed by Fotheringham (1910) and in a revised form by Maunder (1911) and more recently by the Indians (Indian Version, 1979). Fotheringham made use of Schmidt's long-term observational data on new Moon's visibility at Athens (Schmidt, 1868; Mommsen, 1883). More recently, Bruin (1977) developed an independent theoretical criterion, in a way similar to the earlier Hindu and Muslim criterion but making use of modern data for input variables like the brightness of the evening sky, contrast for human eye, etc. Ilyas (1981) intercompared the two independent criteria and found them in excellent agreement after a modification to the Bruin criterion for lowest limit in elongation. Ilyas (1983, 1984a) further studied the specific issue of the lowest limit of elongation (or the Danjon Limit of Lunar Visibility) and developed latitude-season dependent "age" and "moon-set lag" criteria (Ilyas, 1983a, 1985) besides the composite "Δz, a_L" form of the Fotheringham-Maunder criterion (Ilyas, 1981).

In view of the internal consistency of the two independent criteria, it is reasonable to make use of the composite criterion (Ilyas, 1981) for long-term global computations of new Moon's earliest visibility. However, when these computations were performed (see Ilyas, 1984), it was found that on many dates (especially at high latitudes) the azimuth separation (and elongation) was much larger than the 20° limit (see Figs. 1 and 2) and a simple extrapolation of the criterion was found to lead to unacceptable results. It was therefore necessary to examine the extrapolation of the criterion to large azimuth-separation (and elongation) as well as the limiting altitude-separation. The results of this analysis are the subject of this paper.

2. Analysis and results

The Fotheringham, Maunder and Indian Versions of the criterion (Ilyas, 1978) and the original data used by Fotheringham (see Ashbrook, 1971) are shown in Figs. 1 and 2. We notice that all the three curves (Fig. 1) are limited to about 20° in azimuth-separation (ΔAz) and give no indication as to what should be done for larger ΔAz. Beyond 20°, there are two possible extensions – a simple extrapolation of the curve and a horizontal limiting altitude curve. These situations are shown by the broken curves in Fig. 3. Indeed, on the basis of the linear (downward) extrapolation (Fig. 3, lower broken curve), Ilyas' inverted "Δz, a_L" data

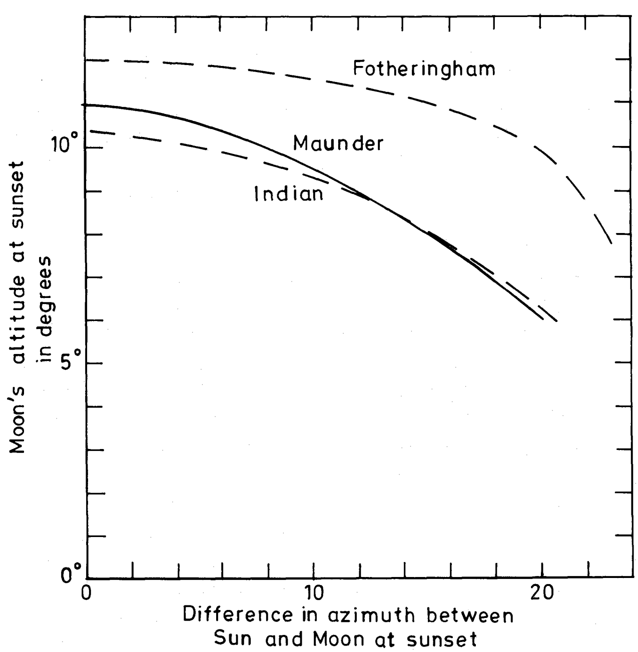

Fig. 1. The Fotheringham (1910) criterion in original (see Fig. 2) and revised (Maunder and Indian) forms

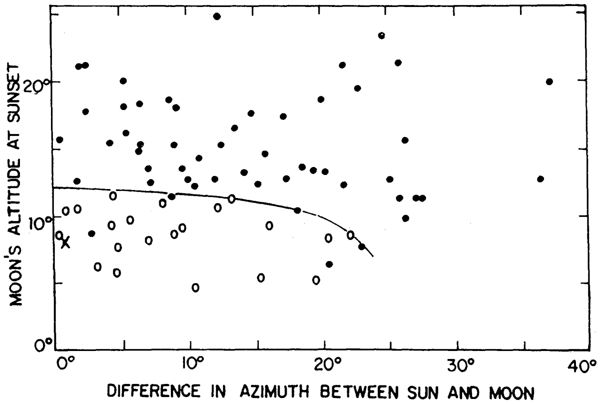

Fig. 2. The Schmidt's data on new moon's earliest visibility presented in "Δz, ΔAz" form by Fotheringham (1910). The open circles represent a sighting attempt with negative result whereas a dot represents a sighting. The line provides a demarcation line for sighting criterion

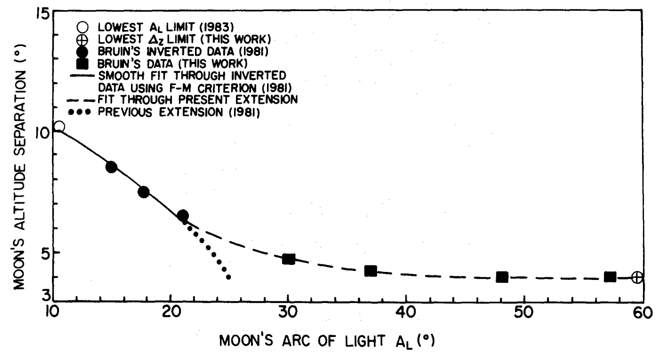

Fig. 4. Development of the "altitude-arc of light" criterion by Ilyas (1981–1988): ○: Ilyas (1981), ●: Ilyas (1981), ■ and ⊕: extension by Ilyas (1984) and this work; •••: Ilyas (1981) extension

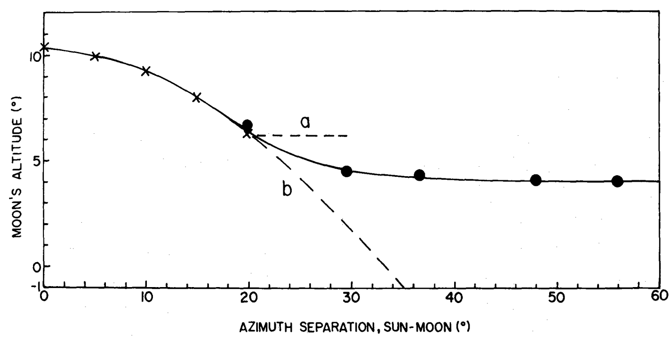

Fig. 3. The extension of the "azimuth-altitude" criterion: ×: criterion already available, ●: present extension using the extension in Fig. 4; curves a and b: simple extrapolations (see text)

(Ilyas, 1981) indicated a corresponding extrapolation beyond 20°. Although from Fig. 3, one would tend to use the lower curve more than the upper one, it may be noted that in this way, at very large ΔAz the extrapolation leads to negative altitude separation. Since the criterion was never before actually put to use for calculations especially at high latitudes – where large ΔAz situations prevail – the necessity of studying the criterion at large ΔAz did not really arise prior to this study.

Schmidt's observational data from which the composite criterion was developed (see Fig. 2) is not of much help at large azimuth separations because Schmidt's location (Athens) was not suitable to obtain data under "low altitude-large azimuth separation" condition applicable to the high latitudes. However, Bruin's visibility curves applicable to large crescent width, ω (hence large a_L) can be used to extend the "Δz, a_L" criterion to large elongations in a way we did for smaller widths (Ilyas, 1984). For different crescent widths, ω, the minimum values of "h + s" were determined which are shown in Fig. 4. In view of the internal consistency between the inverted data using F – M (Indian Version) criterion and Bruin's at smaller elongations (Ilyas, 1981) (Fig. 4), it is possible to invert the "Δz, a_L" criterion into "Δz, ΔAz" criterion for $\Delta Az > 20°$ using the basic trignometric relations (Appendix 1). The resultant extension curve is shown in Fig. 3.

3. Discussion

We notice that the resultant curve lies between the two extrapolations (a and b) (Fig. 3) and clearly demonstrates that the use of either of them would be erroneous – one overestimating and the other underestimating. We can also note that the effect of larger azimuth separation (or elongation) beyond about 35° becomes increasingly small. The limiting altitude separation (Figs. 3 and 4) is found to be close to 4°. This together with the lowest limit of a_L (10°.4) and ΔAz (0°) fixes both ends of the criterion in either forms (Fig. 5).

Ilyas recently presented long-term (50 years) global visibility data (Ilyas, 1984). Eventhough, the present details on the criterion extension were not given there (Ilyas, 1984) – except its brief mention in the "Δz, a_L" form – the data was nevertheless based on the extended (Δz, a_L) criterion and thus remain valid.

In passing it may be mentioned that an inspection of Danjon's data in his papers (Danjon, 1932, 1936) especially the latter one which contains much more data points, makes the earlier conclusion (Ilyas, 1983) on the limiting elongation to be 10°.5 even stronger. At the time of the previous study (Ilyas, 1983, 1984a) only Danjon's work reported by Ashbrook (1972) was available. The data in the original 1936 paper (his Fig. 29) indicates a much more steeper rise of the "deficiency arc" curve as well as the observational data at smaller elongations being rare. Apparently, this is due to the observations of the new crescents (and deficiency arc) at smaller elongations – close to the limiting value, 10°.5 – being more likely at the lower/tropical latitudes rather than at the mid latitudes (as in case of Danjon's location). This point must be useful in future work on observations of "deficiency arc" in the crescent lengths in the vicinity of limiting elongation.

The criterion presented in Figs. 3–5 represents the most complete and comprehensive modern version (in the two alternate forms) to-date for an unaided (average) human eye under average atmospheric conditions.

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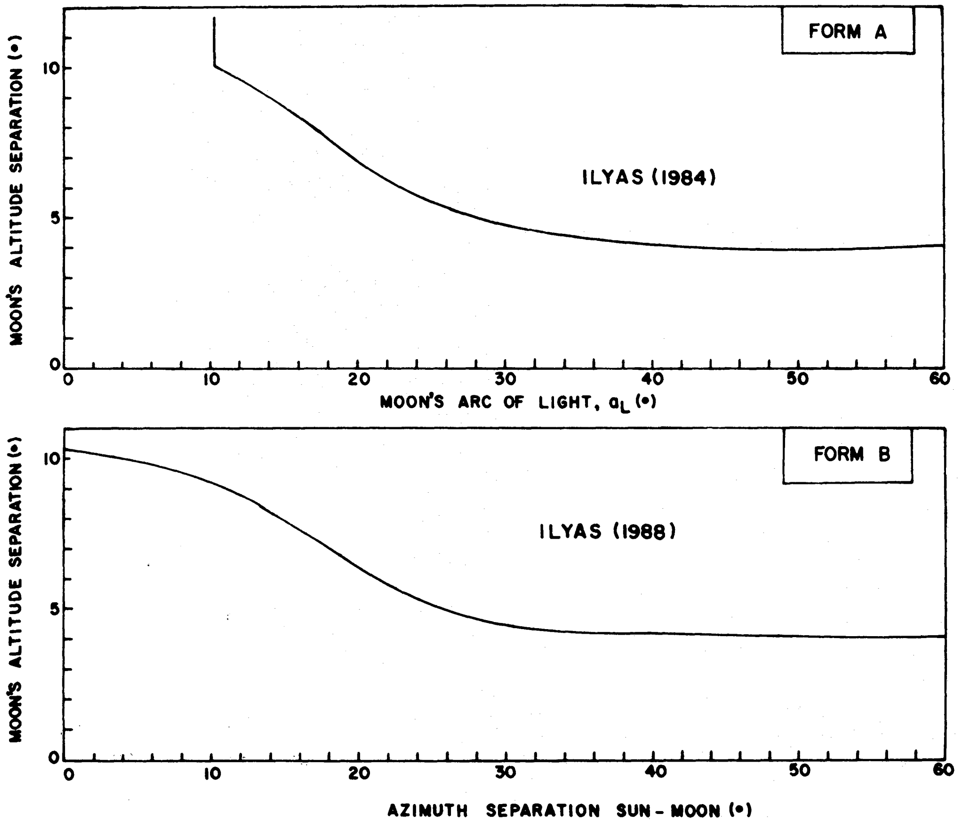

Fig. 5. The composite and updated criterion in two modern forms (Ilyas, 1984, 1988: this work)

Appendix

Converting "Δz, a_L" to "Δz, ΔAz" criterion

If A_1, Az_1 and A_2, Az_2 are altitude and azimuth angles of the Sun and Moon respectively and a_L is the "arc of light" or "elongation" then,

$$\cos(a_L) = \sin A_1 \sin A_2 + \cos A_1 \cos A_2 \cos(Az_2 - Az_1).$$

Since we evaluate the visibility condition at (local) sunset,

$$A_1 = 0;\ \sin A_1 = 0;\ \cos A_1 = 1;\ A_2 = \Delta z;\ Az_2 - Az_1 = \Delta Az.$$

Hence

$$\Delta Az = \cos^{-1}\left(\frac{\cos(a_L)}{(\cos(\Delta z)}\right).$$

References

Ashbrook, J.: 1971, *Sky Telesc.* **42**, 78–79
Ashbrook, J.: 1972, *Sky Telesc.* **43**, 95–96
Bruin, F.: 1977, *Vistas Astron.* **21**, 331–358
Danjon, A.: 1932, *L'Astronomie* **46**, 57–66
Danjon, A.: 1936, *Bull. Soc. Astron. France* **50**, 57–65
Fotheringham, J.K.: 1910, *Monthly Notices Roy. Astron. Soc.* **70**, 527–531
Ilyas, M.: 1978, *J. Mal. Bran. Roy. Asiatic Soc.* **51(2)**, 56–68
Ilyas, M.: 1981, *Quart. J. Roy. Astron. Soc.* **22**, 154–160
Ilyas, M.: 1983, *J. Roy. Astron. Soc. Canada* **77**, 214–219
Ilyas, M.: 1983a, *Observatory* **103**, 26–28
Ilyas, M.: 1984, *A Modern Guide to Astronomical Calculations of Islamic Calendar, Times & Qibla*, Kuala Lumpur, Berita
Ilyas, M.: 1984a, *Quart. J. Roy. Astron. Soc.* **25**, 421–422
Ilyas, M.: 1985, *J. Brit. Astron. Assoc.* **95**, 183–184
Ilyas, M.: 1986, in *History of Oriental Astronomy, IAU Coll.* **91**, Cambridge University Press, Cambridge, pp. 147–152
Indian Version: 1979, *Indian Astronomical Ephemeris*, India Meteorology Dept., New Delhi
Maunder, E.W.: 1911, *J. Brit. Astron. Assoc.* **21**, 356–362
Mommsen, A.: 1883, *Chronologie*, Hilprecht University
Schmidt, J.: 1868, *Astron. Nachr.* (*No. 1693*) **71**, 201–208