# THE DANJON LIMIT OF LUNAR VISIBILITY: A RE-EXAMINATION

BY MOHAMMAD ILYAS
*Penang, Malaysia*

*(Received February 14, 1983)*

ABSTRACT

It is found that the correct value of the Danjon's limiting elongation for lunar visibility is close to 10°5. The limit provides only a general guide, not sufficient for calendrical purposes.

*Introduction*. The earliest visibility of the new moon is of considerable calendrical importance (especially for the Judaic and Islamic calendars) and has formed the subject of astronomical studies from earliest times (Ilyas, 1981a). From very early times, it has been known that, after conjunction, the new moon needs to grow to some extent before it can become visible. This minimum requirement for earliest visibility has been represented in various astronomical terms including moon's age (Ilyas, 1983) and moonset lag (Ilyas, 1981a). Two relatively comprehensive forms of the earliest lunar visibility involve elongation (arc of light, $a_L$) and azimuth separation of the moon, both as a function of moon-sun altitude separation at local sunset respectively (Ilyas, 1981b; 1982a).

The limiting value of elongation for earliest visibility, by itself, is also of interest. This is also known as the Danjon Limit after Danjon who first tried to study the physical cause for this (Danjon, 1932). Recently, in a calendrical work, the Danjon Limit was mistakenly used as a rigorous basis (Calendar Commission, 1978) giving rise to conflicting results. This necessitated a close examination of the Danjon Limit specially with reference to its calendrical predictive use.

*The Danjon Limit*. Danjon is reported to have noticed the phenomenon of crescent's limb shortening for a young moon in 1931 (Ashbrook, 1972) which he later discovered to be a general phenomenon. Danjon (1932) is quoted by Ashbrook as having explained the phenomenon as being due to the shadowing effect of lunar mountains at small elongations. Furthermore, Danjon deduced the magnitude of this crescent shortening in the form of a deficiency arc as a function of elongation. For this he employed measurements of estimated crescent length and elongation. The result is redrawn in figure 1. Using the curve drawn through the scattered data points, Danjon argued that the (extrapolated) curve shows a deficiency of 7° at an elongation of 7°, hence the 7° elongation is the limiting value for crescent's visibility.

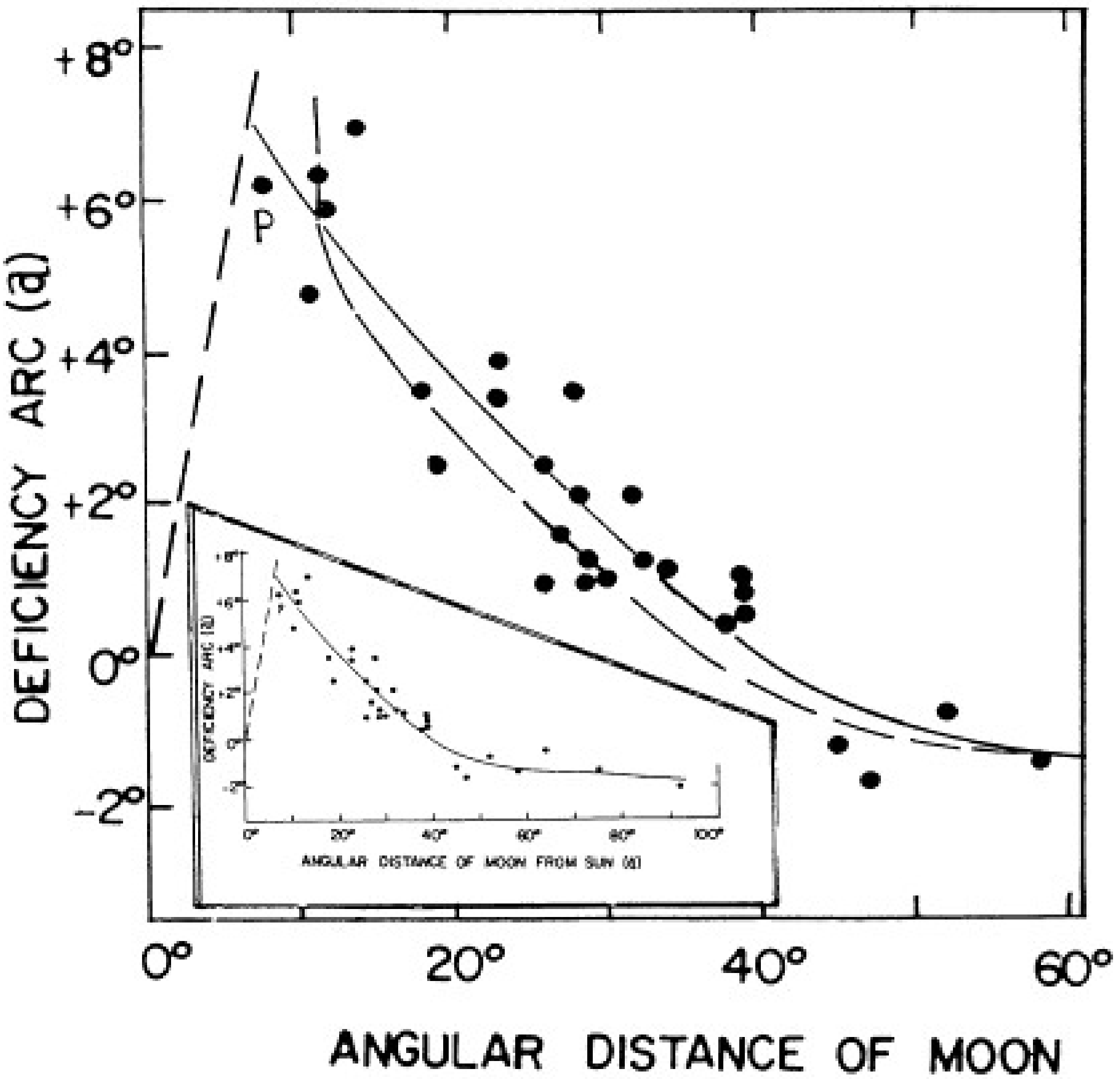


FIG. 1—In this (Danjon's) diagram, the dashed line indicates where the horizontal and vertical scales are numerically equal. The line meets the extrapolated curve through the data as drawn by Danjon at 7° elongation. The curve however may be easily modified especially at the small elongation side if the weight of the first (single) point, *P*, is reduced (for discussion see text).

*Examination of the Danjon Curve*. Danjon's data are reproduced in figure 1 (from Ashbrook, 1972). It is clear that there is a considerable scatter in deficiency-arc data in the vicinity of 30° elongation (also there is some uncertainty associated in the estimations for each data point). Although one could draw the curve downward giving more weight to predictions from the lower data points, in the present context, it is not very important to do so. The region around 10° elongation is, however, critical in determining the limiting value. Unfortunately, the data in this region are scarce – just five points. We may note that as we approach the 10° elongation, the data points are scattered vertically. More importantly, there is *just one observed datum point in the entire set* obtained for an elongation less than 10°.

If this single datum is neglected or given a lower weight, the behaviour curve would have a steep rise at about 10° elongation. Unfortunately, Danjon had used this single data point in extrapolating his curve. This resulted in the estimate of the limiting value of elongation for visibility as being 7° rather than about 10°. This may not be a serious problem if his estimate is used as a general guide within the experimental limitations. However, this difference may lead to gross errors if it is mistaken as a basis for refined calendrical work.

*Other Evaluation of the Limiting Elongation*. In a recent study (Ilyas, 1981b), it was found that Bruin's theoretically based earliest-visibility criterion extended to smaller elongations was in excellent agreement with the purely observational criterion of Maunder. The latter was computationally inverted from the ($\Delta Z$, $\Delta AZ$) from to ($\Delta Z$, $a_L$) form for intercomparison. An examination of the inverted data (figure 2) leads to the lowest possible limit of elongation ($a_L$ or $e$) as being about 10°.5 (the observation limit may be directly evaluated as shown in the appendix).

The composite ($\Delta Z$, $a_L$) criterion was recently used to develop latitude-season dependent 'moon's age' and 'moonset lag' criterion (Ilyas, 1983a; Ilyas 1983b). In both cases, the inverted data, including the limiting values, are found to be consistent with the past observational records and experience thus giving great confidence in this limiting value. This is also consistent with the general ruling from RGO (1974), "It is unlikely that the new crescent will be visible unless the elongation exceeds 10° and the altitude of the moon exceeds 5° when the depression of the sun is 3°".

*Discussion of the Danjon Limit*. Bruin's visibility criterion is based on physical considerations as model input. These are the brightness of the evening sky, intensity of the lunar crescent (assuming smooth surface) and contrast requirements for the human eye. On the other hand, the criterion of Maunder and Fotheringham is based directly on actual visibility data. The internal consistency of the two independent systems (Ilyas, 1981b) is very good and the limiting minimum-elongation value for crescent visibility from these is practical and accurate.

From a re-examination of Danjon's data, it may be safely concluded that a more appropriate limiting elongation is closer to this value, i.e. $\simeq$10°.5.

The actual value aside, what Danjon had proposed is obviously an approximate minimum elongation necessary for the crescent's earliest visibility, to serve as a general guide. Just by itself, the limit may not be a sufficient condition. Therefore, even the (revised) Danjon Limit alone, cannot be used as a basis for calendrical visibility prediction. Unfortunately, this critical difference seems to have been overlooked at the 1978 calendrical conference held at Istanbul which adopted too

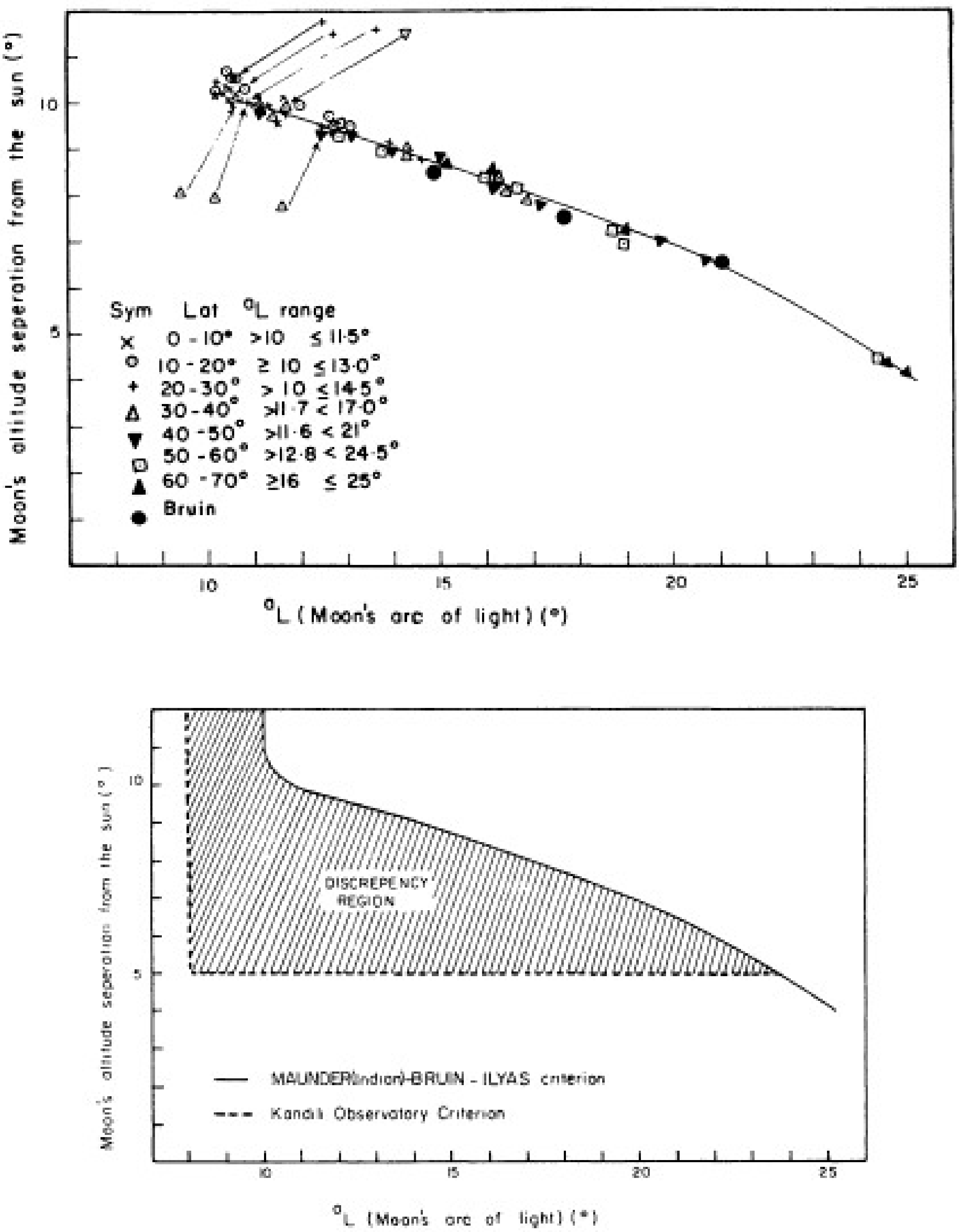


FIG. 2—(a) Variation of the moon's altitude separation with elongation for meeting the minimum visibility requirements according to Maunder and Bruin, showing the lowest limit to be about 10°.5 elongation (note the accumulation of data in this region). (b) Comparison of the composite criterion with the underestimated and over simplified "8° and 5°" criterion employed by the Calendar Commission based on the Danjon Limit. The latter would indicate the earliest visibility longitudes ($\lambda_0$) to the east by 60°–80° in longitude at tropical latitudes.

simple ($a_L = 8°$, $\Delta Z = 5°$) a criterion by joining, rather arbitrarily the Danjon Limit (8°) with 5° altitude separation (Dizer, 1980) as discussed elsewhere (Ilyas, 1981b, 1982a). Its effect on the predicted data, especially in the tropical region, would become more serious due to the underestimation in the limiting value (Ilyas, 1982b).

*The Shadowing Effect*. Besides the evaluation of the limiting elongation, Danjon also tried to understand the cause of the limit by way of limb shortening. He explained the phenomenon as being due to a shadowing effect of the lunar mountains. The shadow cast on the sunlit part facing the earth resulted in a deficiency arc and, at the limiting elongation, the deficiency arc equalled the elongation leaving no actual sunlit part facing the earth. Limb shortening is an observed phenomenon. However, from a practical point of view (crescent's earliest visibility), the phenomenon may not be important. This is because the limiting elongation in the Bruin-Maunder criterion is essentially based on the intensity requirements (see Bruin, 1977). At elongations greater than the limiting value, the effect becomes insignificant as reflected in the Danjon diagram of the deficiency arc, in which the redrawn curve has a steep rise at about 10° elongation. This is also supported by the fact that the limiting elongation from the Bruin-Maunder criterion is essentially based on the disk intensity and contrast requirements (see Bruin, 1977).

*Conclusion*. The value of the limiting elongation for earliest lunar visibility, also referred to as the Danjon Limit, is about 10°.5 rather than Danjon's earlier extrapolated value of 7°. The Danjon Limit provides a minimum but not a sufficient condition for earliest lunar visibility and requires use of another positional parameter (altitude separation) in a usual lunar-visibility prediction criterion.

*Acknowledgement*. Thanks are due to professor M. Dizer (Kandilli Observatory) for providing information on the Calendar-Commission criterion. Some of the literature pertaining to the subject was made available by the International Centre for Theoretical Physics (ICTP), Trieste, under a Junior Associateship program.

*Mohammad Ilyas,*
*School of Physics,*
*University of Science of Malaysia,*
*Penang,*
*Malaysia*