The Visibility of the Lunar Crescent.

In the *Classical Quarterly* for July and October, 1920, xiv. p. 172, Dr. Holmes contributed a note on the earliest visible phase of the Moon, in which he referred to my paper on the subject in *Monthly Notices*, lxx. pp. 527–31 (1910). In that paper I analysed Schmidt's observations, made with two exceptions at Athens, and tabulated the altitude of the Moon and difference in the azimuths of Sun and Moon at sunset, ignoring parallax and refraction in the case of both luminaries. The observations thus treated showed a clear dividing-line between visible and invisible phases of the Moon. Dr. Holmes, who has specially in view the visibility of the crescent of −57, March 25, at Geneva, raises a doubt whether a rule based on observations in the clear atmosphere of Athens would apply in Switzerland, and asks the question, "Has Dr. Fotheringham or any other trustworthy observer ever seen with the naked eye in an atmosphere no clearer than that of Geneva a moon not more than 27 hours old?"

This question is easily answered. I have collected from the *Journal of the British Astronomical Association* seven such observations made in Great Britain, and Herr Schoch, a persistent observer of such phenomena, has sent me three such observations made by himself in Germany and one made by himself in Belgium. He also arranged for an international search for the crescent of 1921 February 8, and obtained reports from numerous observers in Germany, Italy, Spain, and Portugal. At his request, and with the aid of Prof. Turner, I arranged for a watch to be kept for the same crescent at numerous places in England and the United States. The following places reported that the Moon was obscured by clouds, fog, or thick haze:—Heidelberg, Mannheim, Ludwigshafen, Milan, Florence, Rome, Naples, Barcelona, Madrid, Charing, Oxford, Washington, Mount Hamilton, San Francisco. At San Fernando in Spain and at Lisbon the sky was clear, but the Moon was not seen. At Mount Wilson the Moon was observed, but a telescope was used as a pointer. At the Harvard College Observatory, Cambridge, Massachusetts, at the Lowell Observatory, Flagstaff, Arizona, and at the Steward Observatory, Tucson, Arizona, the Moon was observed without instrumental aid. My selection from the *J. B. A. A.* includes only evening observations, and only observations made without instrumental aid. In the accompanying table, I tabulate these observations and add the lunar altitude and difference in azimuth at sunset, showing how far the Moon lay above or below my dividing-line. For observations (1) and (2) I have taken these figures from Mr. Maunder's paper, *J. B. A. A.* xxi. p. 358 (1911). For the other observations I have made the necessary computations myself. I add beneath a line the figures, so far as computed, for the unsuccessful observations at San Fernando and Lisbon.

Place.	Date.	G.M.T.	Observer.	Moon's Age.	Authority.	Altitude at Sunset.	Difference in Azimuth at Sunset.	Distance above dividing-line.
		h m		h m		°	°	°
(1) 12 miles west of Manchester.	1864 Feb. 8.	5 40	Rev. S. J. Johnson.	23 30	*J. B. A. A.* xi. 277 (1901).	13·8	4·5	+1·9
(2) Bristol	1875 June 4.	9 10	W. F. Denning.	22 49	*J. B. A. A.* xix. 242 (1909).	11·8	7·1	+0·1
(3) Bristol	1881 March 30.	7 10	W. F. Denning.	20 38	*J. B. A. A.* xix. 242 (1909).	11·4	1·2	−0·6
(4) Berlin	1899 March 12.	5 37	C. Schoch.	21 44	Letter.	12·9	1·5	+0·9
(5) Edinburgh	1908 Feb. 3.	5 0	A. C. Henderson.	22 30	*J. B. A. A.* xix. 255 (1909).	11·0	16·1	+0·2
(6) Heidelberg	1915 March 16.	6 1	C. Schoch.	22 16	Letter.	10·8	1·6	−1·2
(7) Heidelberg	1916 April 3.	6 25	C. Schoch.	26 6	Letter.	13·8	1·4	+1·8
(8) Scarborough ...	1916 May 2.	8 0	Lizzie King and Nellie Collinson.	14 30	*J. B. A. A.*, xxvii. 36 (1916).	8·3	0·6	−3·7
(9) Heighington ...	1916 May 2.	8 15	Mrs. Willimott and her daughter.	14 45	*J. B. A. A.* xxvii. 36 (1916).	8·3	0·7	−3·7
(10) Ciney	1918 March 13.	6 8	C. Schoch.	22 17	Letter.	13·8	1·5	+1·8
(11) Leeds	1919 April 1.	7 10	C. T. Whitmell.	22 0	*J. B. A. A.* xxix. 141 (1919).	12·6	3·7	+0·7
(12) Cambridge, Mass.	1921 Feb. 8.	10 42	Leon Campbell and Miss Harwood.	22 5	Letter.	10·8	0·4	−1·2
(13) Flagstaff	1921 Feb. 8.	13 21·5	Mr. Lampland and Dr. V. M. Slipher.	24 45	Letter.	12·0	0·6	0
(14) Tucson	1921 Feb. 8.	13 30	Godfrey Sykes.	24 53	Letter.	11·9	1·3	−0·1
San Fernando...	1921 Feb. 8.	—	Not seen.	17 30	Letter.	Not computed.	Not computed.	Not computed.
Lisbon	1921 Feb. 8.	—	Not seen.	18 0	Letter.	9·1	1·1	−2·9

It will be noted that the observers named have not merely done as well as Schmidt at Athens, but in certain cases they have done better. Observations (8) and (9), made in ideal weather conditions, can hardly be regarded as typical of the possibilities of this country. Herr Schoch has constructed tables, as yet unpublished, for the determination of the earliest visibility of the crescent to the naked eye. He writes that all the observations shown above, except (2), (8), and (9), accord with these tables. He states further that in observations (4) and (10), separated by one nineteen years' period, the Moon was so clearly seen that there can be no doubt that she would have been visible, even if new Moon had occurred one hour later, and he enunciates the rule that from February 1 to April 15 the minimum age of the Moon for visibility in terrestrial latitudes between 45° and 52° varies from 20 hours to 23 hours; the former of these values should hold where the Moon's mean anomaly is between 340° and 20° and her argument of latitude between 70° and 110°, the latter when her mean anomaly is between 160° and 200°, and her argument of latitude between 250° and 290°.

Observation (12) is of considerable interest, since the mean atmospheric absorption of light is, to the best of my knowledge, greater at Cambridge, Mass., than at any other place for which it has been determined, with the single exception of Catania. On this occasion the Moon was clearly on the limit of visibility, since two observers saw the crescent and two failed to detect it.

Perhaps a little explanation of the reason why, so long as the particular part of the sky is not obscured by clouds or thick haze, differences in atmospheric conditions are comparatively unimportant may be useful. The apparent brilliance of a heavenly body depends on the quantity as well as the quality of the atmosphere through which its light passes. The atmospheric absorption of light increases as the body descends towards the horizon or as the observer descends towards sea-level. Thus the mean co-efficient of absorption at the observatory on Mount Sentis in Switzerland, 8215 ft. above sea-level, has been found to be 0·14*. If we exclude mountain observatories on the one side and the uniquely unfavourable atmosphere of Catania on the other, that co-efficient varies in the Old World between the two extremes of 0·187 at Cairo and 0·253 at Oxford. Allowing for the difference in altitude between the Sentis and Geneva, which stands 1243 ft. above sea-level, we should expect the mean co-efficient at Geneva to be 0·18. In other words, the brilliance of a heavenly body in Switzerland at the altitude of Geneva should be slightly greater than that of the same body at the same altitude above the horizon when observed from a small

* Müller, "Photometrische und spectroskopische Beobachtungen auf dem Säntis," *Publicationen des astrophysikalischen Observatoriums zu Potsdam*, viii. (1893).

altitude above sea-level in the most favourable climate of the Old World. Now, an increase in the age of the crescent increases not only its phase, but also its altitude at the time of the evening when it is looked for. As an example, I have compared the Scarborough observation (No. 8) with an imaginary observation made one hour later in the same latitude, but 15° farther west. In this way I am able to add an hour to the Moon's age without altering the local time. According to Lambert's theory, the brilliance of a small moon should vary as the cube of the supplement of its phase-angle, and Dr. Russell * has shown that this ratio holds good of all but the largest phases, so far as observation has been possible. Assuming that the same ratio holds good for the early phases for which it has not been tested by observation, I find that in the selected example the addition of one hour to the Moon's age would increase her brilliance by nearly 0·17 of a stellar magnitude. Assuming, again, for that remarkable evening a co-efficient of atmospheric absorption of 0·18, I find that the increase in the Moon's altitude resulting from the increase in age would increase her brilliance by nearly 0·15 of a stellar magnitude, so that the total increase in brilliance resulting from an addition of one hour to her age would amount to 0·31 of a stellar magnitude. Now, if, instead of increasing the Moon's age, I diminish the co-efficient of the atmospheric absorption of light by an amount equal to the difference between the observed co-efficients for Oxford and Cairo, I find that I shall increase her apparent brilliance by 0·83 of a stellar magnitude. It follows that in this particular case the difference between the atmospheres of Oxford and Cairo would affect the brilliance, and, therefore, presumably, the visibility of the Moon, by only 2·7 times as much as the addition of one hour to her age. It would be very rash to generalize from this one example. I quote it merely as an indication of the relative importance of phase and clearness of atmosphere.

I may add that if observations are to be of any use for the interpretation of references to the first appearance of the Moon or of dates dependent on that appearance, it is essential that the Moon should not be discovered either with a pointer or with any kind of optical glass, even if she is seen with the naked eye after being so discovered. The ancient observer had to dispense with all optical aid, and, except perhaps at Babylon, did not know the precise point in the sky where the Moon was to be looked for. On the other hand, he was generally more practised than any of his modern competitors, except perhaps Mr. Denning and Herr Schoch, in picking up heavenly bodies with the naked eye.

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* *Astrophysical Journal*, xliii. p. 114 (1916).